Chapter 1

Introduction

In the natural world, there are some magnetic system with competing interaction. The problem of magnetic properties in this system has been paid attention to.

Magnetic system with competing interaction have been first investigated four decades ago. Well-known examples include the Ising model on the antiferromagnetic triangular lattice studied by G. H. Wannier in 1950 [1] and the Heisenberg helical structure discovered independently by A. Yoshimori and J. Villain in 1959[2]. However, extensive investigations on magnetic systems with competing interactions have really started with the concept of frustration introduced almost at the same time by G. Toulouse and J. Villain in 1977[3] in the context of spin glasses.

The magnetic properties in this system was investigated by many experiments. Then, this curious characteristic, which is not understood by former theories, are found in this system. So, it is interesting to make theoretical studies in this system. Many properties in this systems are still not well understood at present.

In view of the competing interaction effect, we study the static and dynamical magnetic properties. This system has the various phase depending on the competition of different kinds of interaction.

Moreover, the magnetic system with competing interaction have a feature that the frustration is generated by the competition of different kinds of interaction and/or by lattice geometry. As a result, in the case of frustrated spin systems, the ground state configuration is unstable.

We study on the magnetic properties, which are attributed to the competition with the magnetic interaction. We investigate two case about it, which is mentioned below. The real system for the second case model is the perovskite manganese oxide LaMnO$_3$.

1. The two-dimensional square frustrated quantum antiferromagnet Heisenberg system

As the first model with the competition the magnetic interaction, we consider the two-dimensional(2D) square frustrated quantum antiferromagnet
Heisenberg (S=1/2), which contains up to third neighbor couplings (J₁-J₂-J₃ model, see Fig.1.1)[4]. The Hamiltonian is written by

\[ H = J_1 \sum_{(ij)} S_i \cdot S_j + J_2 \sum_{(il)} S_i \cdot S_l + J_3 \sum_{(im)} S_i \cdot S_m, \]

where \( J_1, J_2, \) and \( J_3 \) are the nearest, next-nearest, and third-nearest-neighbor coupling constants, respectively, and all of \( J_i(i = 1, 2, 3) \) are considered to be positive, then and antiferromagnetic. The notations \( (ij), (il) \) and \( (im) \) denote \( \text{nn}, \text{nnn}, \) and \( \text{nnn} \) pair of spins, respectively.

In this system, the zero temperature phase diagram determined by this theory presented distinct four phases depending on the parameter \( j_i = \frac{J_i}{\tilde{J}}: \) \( \text{Néel}, \text{Collinear}, \) and two helical configuration \( H_1 \) and \( H_2, \) and also suggested the existence of spin disordered region over the border-line among different phases due to frustration.

In this system, there are a feature that the frustration occur under a certain combination of couplings. Frustration in magnetic systems is well known to be responsible for a number of curious phenomena. One of the more interesting features is the possible existence of the spin liquid, in which long-range order at absolute zero is destroyed when the frustration effect combine with quantum effect(S = 1/2) which are significant in the low-dimension(2D). Besides, the problem of the disordered frustrated spin system becomes one of the most important issues recently because of its close relation to high-\( T_c \) super conductor[5].

In this thesis, we investigate the effects of spin quantum fluctuation and frustration on spin dynamic properties at the critical temperature, as well as static one, especially in such disordered phases.

The studies of \( \chi(k), S(k) \) and \( F(k, \omega) \) with arbitrary \( k \) and \( \omega \) are expected to shed a light on the problem of the frustrated spin systems. We calculate these functions by high temperature series expansion method. Moreover, we investigate the the frustrated models about both classical(\( S = \infty \)) and quantum(\( S = 1/2 \)) and discussion the quantum effect.

2. The Perovskite Manganese Oxide system(LaMnO₃)

As the second model with the competition the magnetic interaction, we consider the quantum Heisenberg model for the mother compound LaMnO₃, which contains spin (\( S = 1/2 \)) of \( e_g \) electrons on Mn site putting together the three-dimensional(3D) cubic lattice.

\[ H = -J_1 \sum_{(xy)} S_i \cdot S_j - J_2 \sum_{(z)} S_i \cdot S_l. \]

\[ J_1 = \frac{1}{2}(|J_{xy}| - |J_{AF}|). \]
Figure 1.1: (1) square lattice (2) cubic lattice

\[ J_2 = \frac{1}{2}(|J_2| - |J_{AF}|), \]

where \( J_1 \) and \( J_2 \) are the nearest neighbor coupling constants on \( xy \) plane and one along \( z \) direction, respectively (we call this system \( J_1-J_2 \) model in this thesis, see Fig.1.1). The notations \((ij)\) and \((il)\) denote nn pair of spins on \( xy \) plane and one along \( z \) direction, respectively.

\[
J_{xy} = J \left( \frac{3}{2} T_i z T_j z + \frac{1}{4} T_i - T_j^+ - \frac{1}{2} \right) \\
J_z = J \left( \frac{1}{2} T_i - T_j^+ - \frac{1}{2} \right). 
\]

As seen, the resulting Hamiltonian consists of an anisotropic ferromagnetic(F) coupling \( J_{xy(z)} \), which are yield from the pseudospin operator \( T \) for the orbital degree of freedom, and an isotropic antiferromagnetic(AF) coupling \( J_{AF} \). This effective Hamiltonian shows the competition between the anisotropic coupling term \( J_{xy(z)} \) and the isotropic antiferrocoupling \( J_{AF} \) term. In other word, this study is the competing interaction system. Let`s examine the effect of competition in this system. By utilizing the similar Hamiltonian, it is known that there are F-, A(layer-type AF)-, C(chain-type AF)-, and G( isotropic AF)-type with this competition[7].

This is also the our effective Hamiltonian for Mn ion of Mott insulator LaMnO\(_3\). It is shown that the A-type (anisotropic AF) structure[6] is stabilized due to contribution from competing between anisotropic ferromagnetic interaction \( J_{xy(z)} \) and isotropic antiferromagnetic interaction of \( J_{AF} \).

Besides, it is suggested that, owing to the A-type structure, the investigation about the interplay of spin and orbital is important. It is also
significant that we investigated the interplay of spin and orbital in this system in order to investigate A-type structures.

We consider of the our original effective Hamiltonian $J_1$-$J_2$ model by using the local magnetic easy axes. We focus our attention on the spin phase diagram and spin correlation function $S(k, \omega)$, especially at the effect of competing interaction, by means of two-time Green’s functions within Hartree-Fock approximation.

What sort of magnetism has occur depending on interactions? In this thesis, we study and discuss it about two cases by estimating spin correlation function $S(k, \omega)$ and the susceptibility $\chi(k, \omega)$, which are related to neutron scattering.

The thesis is organized as follow: In Chapter 2, we investigate the first case, static and dynamical properties of the frustrated two-dimensional square quantum Heisenberg antiferromagnet. In Chapter 3, we investigate the second case, the magnetic properties in the perovskite manganese oxide. Chapter 4 is devoted to the summary and discussions about this thesis.
Bibliography


