<table>
<thead>
<tr>
<th>項目</th>
<th>内容</th>
</tr>
</thead>
<tbody>
<tr>
<td>Title</td>
<td>A study of hadron production via two-photon process and a measurement of the photon structure function $F^\gamma(X,Q^2)$ at TRISTAN</td>
</tr>
<tr>
<td>Author(s)</td>
<td>Muramatsu, Kanako</td>
</tr>
<tr>
<td>Citation</td>
<td>奈良女子大学博士論文、博士（理学）、博課 甲第38号、平成5年3月24日学位授与</td>
</tr>
<tr>
<td>Issue Date</td>
<td>1993-03-24</td>
</tr>
<tr>
<td>Description</td>
<td>関連論文：物理学論文集 Vol. 332, pp. 477-487</td>
</tr>
<tr>
<td>Textversion</td>
<td>ETD</td>
</tr>
</tbody>
</table>
A Study of Hadron Production via Two Photon Process and
A Measurement of the Photon Structure Function $F_2^{\gamma}(x, Q^2)$ at TRISTAN

Kanako Muramatsu

Human Life and Environmental Science Course, Graduate School of Human Culture, Nara Women’s University
January 1993

Doctoral Thesis
Acknowledgement

It is my great pleasure to be able to participate the experiments of the TOPAZ group. I would like to thank Professor Noboru Fujiwara, Professor Seishi Noguchi and Dr. Hisaki Hayashii for their adequate advices and warm encouragements. I wish to thank to all members of High Energy Physics Laboratory at Nara Woman’s University for their helps and useful discussions.

I would like to thank Professor S. Kawabata who advised me about my analysis and gave me useful discussions. I am also grateful to Professor T. Nozaki and Dr. T. Sasaki for supplying me the program of the calculation for the single-tagging event cross-section and giving me useful advices. In addition I am glad to acknowlege Dr. M. Ronan who supplied me the Blobel’s unfolding program.

I would like to thank to all TOPAZ collaborators who made this experiment successful. We thank the TRISTAN accelerator staffs for the operation of the strage ring and the technical staffs at KEK for achieving high luminosity. I wish to thank to all graduate students in TOPAZ collaboration . They have been always helpful.

Finally I heartily thank my family for their continuous encouragements and supports.
Abstract

We have studied hadron production via two-photon processes in the reaction $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ using data collected at $\sqrt{s} = 58$ GeV by the TOPAZ detector at the TRISTAN $e^+e^-$ collider. The photon structure function $F_2^\gamma(x, Q^2)$ for a quasi-real photon has been measured in the wide $Q^2$ region from 7 to 700 (GeV/c)$^2$. The $x$-dependence of $F_2^\gamma(x, Q^2)$ at average $Q^2$ values of 9.7 and 87 (GeV/c)$^2$ has been corrected for the effects of experimental resolution and incomplete acceptance. The results are compared with the theoretical predictions based on QPM and QCD. The $Q^2$-dependence of the photon structure function in the intermediate $x$ region ($0.3 < x < 0.8$) is measured to be

\[
< F_2^\gamma / \alpha > = 0.357 \pm 0.021 \text{ at } < Q^2 > = 9.7(\text{GeV/c})^2,
\]
\[
< F_2^\gamma / \alpha > = 0.466 \pm 0.063 \text{ at } < Q^2 > = 87(\text{GeV/c})^2,
\]
\[
< F_2^\gamma / \alpha > = 0.753 \pm 0.363 \text{ at } < Q^2 > = 338(\text{GeV/c})^2.
\]

The results are consistent with the theoretical expectations of QPM and QCD.
Contents

1 Introduction 6

2 Theory of the Photon Structure Function 12
  2.1 Formalism of the photon structure function .................. 13
  2.1.1 Deep inelastic electron-photon scattering ................ 13
  2.1.2 Two photon process with single-tag condition ............ 15
  2.2 The photon structure function in the Quark Parton Model .... 17
  2.3 The photon structure function in Quantum Chromodynamics .... 19
     2.3.1 The Altarelli-Parisi evolution equation for the photon structure function in leading log approximation .......... 20
     2.3.2 The parametrizations of parton distribution in the photon .. 23
     2.3.3 FKP formula ............................................. 25

3 Experimental Apparatus 28
  3.1 TRISTAN accelerators ........................................ 28
  3.2 TOPAZ detector .............................................. 28
     3.2.1 Inner Drift Chamber (IDC) ............................ 31
     3.2.2 Time Projection Chamber (TPC) ........................ 34
     3.2.3 Time of Flight counter (TOF) .......................... 38
     3.2.4 Superconducting solenoid magnet ....................... 38
     3.2.5 Barrel Calorimeter (BCL) ............................. 38
     3.2.6 Barrel Muon Drift Chamber (MDC) ...................... 40
     3.2.7 Lumminosity Monitor (LUM) .......................... 40
     3.2.8 Endcap Drift Chamber (EDC) .......................... 41
     3.2.9 Endcap Calorimeter (ECL) ........................... 43
  3.3 New detectors .............................................. 43
     3.3.1 Vertex Chamber (VTX) .................................. 45
     3.3.2 Trigger Chamber (TCH) .................................. 47
3.3.3 Forward Calorimeter (FCL) ........................................ 47
3.3.4 Ring Calorimeter (RCL) .......................................... 54
3.3.5 Forward-backward Muon Chamber (F/B MU) .................. 55
3.4 Trigger system ......................................................... 55
  3.4.1 Energy Trigger .................................................. 55
  3.4.2 Charged Trigger ................................................ 58
  3.4.3 Software Trigger ............................................... 59
3.5 Data acquisition system .............................................. 61

4 Performance of the Luminosity Monitor 65
  4.1 Trigger system .................................................. 66
  4.2 Performance of counters ......................................... 66
     4.2.1 Pulse height distributions .................................. 66
     4.2.2 Counter stability .......................................... 70
  4.3 Luminosity measurement ......................................... 70
     4.3.1 Bhabha event selection ................................... 70
     4.3.2 Corrections for the luminosity measurement .......... 70
     4.3.3 Systematic error of luminosity measurement ....... 72
     4.3.4 Results of luminosity measurement .................. 78

5 Raw Data Processing 80
  5.1 Flow of Data Processing ......................................... 80
  5.2 Event Reduction ............................................... 82
  5.3 Event Reconstruction ............................................ 82
     5.3.1 Cluster Reconstruction ................................... 82
     5.3.2 Track Reconstruction for Time Projection Chamber .. 85

6 Event Selection 90
  6.1 Event selection of single-tag events ........................ 90
     6.1.1 Selection criteria for single-tag events ............ 91
  6.2 Backgrounds ..................................................... 93

7 Experimental results and comparison with Monte Carlo predictions 100
  7.1 Monte Carlo Simulation ....................................... 100
     7.1.1 Event Generation ...................................... 102
     7.1.2 Detector Simulation .................................... 106
     7.1.3 Trigger simulation .................................... 106
7.1.4 Results of Monte Carlo Simulation .................................. 109
7.2 Background estimation .................................................... 109
7.3 Experimental results ...................................................... 115
7.4 Comparison with theoretical predictions .............................. 117

8 Acceptance Correction  .................................................... 122
  8.1 Unfolding method by V.Blobel ........................................ 123
     8.1.1 Discretization .................................................... 124
     8.1.2 Unfolding without Regularization .............................. 125
     8.1.3 Unfolding with Regularization .................................. 126

9 Results and Discussion .................................................... 132
  9.1 Experimental results ................................................... 132
  9.2 Comparison with theoretical calculations ......................... 133
     9.2.1 Comparison with the Quark Parton Model .................... 135
     9.2.2 Comparison with the QCD parametrization formulae given by
           DG and LAC .................................................... 135
     9.2.3 Comparison with QCD prediction given by FKP ............... 138
  9.3 $Q^2$-dependence of the Photon Structure Function ............... 138

10 Conclusion .............................................................. 142

A Unfolding methods ........................................................ 144
  A.1 Correction factor method ............................................... 144
  A.2 Matrix inversion method ............................................... 145
  A.3 Approximate unfolding ............................................... 145
Chapter 1

Introduction

It is believed that the ultimate constituents of matter in Nature are structureless spin-1/2 particles, leptons and quarks, and gauge bosons with spin-1. The gauge bosons mediate various forces between leptons and quarks. There are three kinds of charged leptons; electron (e), muon (µ) and tau (τ) and three neutral leptons; neutrinos (ν_e, ν_µ, ν_τ). The charged leptons feel the electromagnetic and weak force. The neutral leptons feel only weak force. On the other hand, there are six kinds of quarks called up (u), down (d), strange (s), charm (c) bottom (b) and top (t)\footnote{The top quark (t) has not been discovered yet. Some indirect evidence is reported in the LEP experiment \cite{4}.} The quarks feel strong force in addition to the electromagnetic and weak forces.

The electromagnetic force is described by Quantum Electrodynamics (QED). This force is mediated by exchanging a massless gauge boson (the “photon”). The coupling strength of this force is proportional to the electric charge \( a_{\text{QED}} = e^2/4\pi \sim 1/137 \). The weak force is mediated by exchanging massive gauge bosons, termed the \( Z^0 \) and \( W^\pm \) bosons. The electromagnetic and weak forces were unified by Weinberg and Salam, in the theory called ”Electroweak dynamics” or the ”Weinberg-Salam model”. Recently, this theory has been tested very precisely, and no phenomenon which violates the theory has been observed in the energy region up to 100 GeV.

The theory of the strong force is called Quantum Chromodynamics (QCD). The source of this force is the color charge. Each quark carries color charge that can take on three values, red (R), green (G) and blue (B). Quarks are coupled strongly by gluons which exchange color. Since a gluon itself carries color charges, it can interact with other gluons. The existence of this gluon self-interaction shows strikingly different phenomena from QED. Because of this self-coupling, the effective coupling strength of the strong interaction \( a_{\text{QCD}}(Q^2) \) decrease logarithmically as \( Q^2 \).
increases. As a result, the coupling strength of the strong interaction can be written by
\[ \alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \ln \frac{Q^2}{\Lambda^2}}. \] (1.1)
in leading (\( \ln Q^2 \)) order, where \( n_f \) is the number of flavors relevant to the reaction, \( \Lambda \) is the QCD scale parameter and \( Q^2 \) is a typical energy scale of a reaction. In other words, quarks interact weakly at high energy (short distance), but interact strongly at low energy (large distance). The phenomena at high energy can be calculated in QCD using perturbative methods. However the low energy phenomena, such as bound states of quarks (i.e. hadrons) and low energy hadron interactions cannot be treated by perturbative methods since \( \alpha_{QCD}(Q^2) \) becomes greater than 1. For example mesons (\( \pi, \rho, \omega, \phi, \cdots \)) are bound states of a quark and an antiquark (\( q\bar{q} \)) and baryons (\( p, n, \Lambda, \Sigma, \cdots \)) are three quark (\( qqq \)) bound state, respectively. These bound states can not be treated by perturbative methods. The soft nonperturbative phenomena are usually treated by lattice calculations or phenomenological models.

One of the best examples of high energy (short distance) phenomena is deep inelastic electron-proton scattering. The first evidence of the existence of free point-like constituents inside the proton was found in this reaction [5]. Fig. 1.1 shows this situation schematically. In elastic electron-proton scattering, the low \( Q^2 \) virtual photon emitted from the electron could see only the full size of the proton, because the wavelength of the photon is almost the same size as the proton radius (Fig. 1.1 (a)). Here \( Q^2 \) means the squared mass of the virtual photon. This situation is changed drastically if the energy of the electron increases, and the wavelength of probing photon becomes short enough that the virtual photon with high \( Q^2 \) can resolve point-like constituents, called partons (quarks and gluons), inside a proton (Fig. 1.1 (b) and (c)). In this deep inelastic electron-proton scattering, the photon acts as a very good probe to study the structure of complicated objects like proton, since this high \( Q^2 \) photon makes point-like interactions with the target parton inside a proton.

If a target proton is replaced by a photon, the structure of the target photon may be studied instead of the structure of a target proton. This reaction is called “deep inelastic electron-photon” scattering. It is known that the photon exhibits properties similar to vector mesons, in addition to the well known point-like nature of the photon. Experimentally, the hadronic nature of the photon is observed in low energy photoproduction \( \gamma p \to \rho p \) experiments [6] and \( e^+e^- \to e^+e^- + \text{hadrons} \) reactions. In
Figure 1.1: The schematic view of the electron-proton scattering. The Fig. (a) shows elastic electron-proton scattering, (b) shows deep inelastic electron-proton scattering, where the substructure quarks can be seen in this process. (c) If the $Q^2$ is increased, the gluons in the proton appear.
Figure 1.2: The deep inelastic electron photon scattering in $e^+e^-$ reactions.

the $e^+e^- \rightarrow e^+e^- + hadrons$ reaction, deep inelastic electron-photon scattering can be seen by detecting either of the scattered electrons (or positron) at a large angle and by restricting the other one to be scattered by a small angle, as shown in Fig. 1.2[7]. Since only one electron (or positron) is observed by a detector, this method is called a single-tag experiment. In this single-tag condition, one can study the nature of the photon precisely. This situation is similar to the deep inelastic electron-proton experiment but here the structure of the photon is probed by a high $Q^2$ virtual photon. It is therefore natural to introduce the notation of the photon structure function in analogy to the well known deep inelastic electron-proton scattering.

The photon structure function has been measured in various experiments in $e^+e^-$ collisions, PETRA, PEP and TRISTAN (AMY) [8]. The $Q^2$ region and the average $Q^2$ ($<Q^2>$) of previous experiment are summarized in Table 1.1.

In this thesis we report the study of hadron production in photon-photon collisions using the reaction $e^+e^- \rightarrow e^+e^- + hadrons$ with the single tagging condition. We measured the photon structure function using this process in the wide $Q^2$ region from 7 GeV$^2$ to 700 GeV$^2$. This wide range of $Q^2$ covers the regions where nonperturbative effects are important and also the regions where perturbative QCD dominates. One of the most important subjects in this thesis is to study the transition region between
<table>
<thead>
<tr>
<th>Collider</th>
<th>Experiment</th>
<th>$\sqrt{S}$ (GeV)</th>
<th>Luminosity (pb$^{-1}$)</th>
<th>$Q^2$ (GeV$^2$)</th>
<th>$&lt; Q^2 &gt;$ (GeV$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PETRA</td>
<td>PLUTO</td>
<td>34.6</td>
<td>19.0</td>
<td>0.1 $\sim$ 1.0</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td></td>
<td>34.6</td>
<td>30.1</td>
<td>1.5 $\sim$ 16.</td>
<td>5.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>34.6</td>
<td>40.1</td>
<td>18. $\sim$ 100.</td>
<td>45.</td>
</tr>
<tr>
<td>JADE</td>
<td></td>
<td>33.6</td>
<td>72.5</td>
<td>10. $\sim$ 55.</td>
<td>24.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>33.6</td>
<td>72.5</td>
<td>30. $\sim$ 220.</td>
<td>100.</td>
</tr>
<tr>
<td>TASSO</td>
<td></td>
<td>32. $\sim$ 35.</td>
<td>52.5</td>
<td>7. $\sim$ 70.</td>
<td>23.</td>
</tr>
<tr>
<td>CELLO</td>
<td></td>
<td>34.</td>
<td>9.5</td>
<td></td>
<td>9.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>34.</td>
<td>7.4</td>
<td></td>
<td>100.</td>
</tr>
<tr>
<td>PEP</td>
<td>TPC/2$\gamma$</td>
<td>29.</td>
<td>50.</td>
<td>0.2 $\sim$ 7.</td>
<td>5.1</td>
</tr>
<tr>
<td>TRISTAN</td>
<td>AMY</td>
<td>50. $\sim$ 61.4</td>
<td>33.8</td>
<td>30. $\sim$ 110.</td>
<td>73.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50. $\sim$ 61.4</td>
<td>33.8</td>
<td>80. $\sim$ 320.</td>
<td>160.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50. $\sim$ 61.4</td>
<td>33.8</td>
<td>160. $\sim$ 750.</td>
<td>390.</td>
</tr>
</tbody>
</table>

Table 1.1: The summary of previous single tagging $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ experiments.

perturbative and nonperturbative QCD. In this thesis, the data were corrected for the effects of incomplete acceptance and finite resolution of our detector. We call this as an “unfolding” in this thesis, since the observed distribution is unfolded to a real distribution by correcting the detector effects. The results were compared with various theoretical predictions of the Quark Parton Model (QPM) and QCD.

The data were collected by the TOPAZ detector at TRISTAN for the past five years since April 1987. The center of mass energy was gradually raised from 52 GeV to 62 GeV during the first two years. The accumulated data at TOPAZ detector was about 24.6 pb$^{-1}$, during this period. Since 1990, the center of mass energy was fixed at 58 GeV in order to accumulate higher luminosity. In the high luminosity experiments, several new detectors were installed in the TOPAZ detector in Autumn 1989 to make the TOPAZ detector hermetic. Especially, the Forward Calorimeter has extended the $Q^2$ region of tagged electron from 30 GeV$^2$ to 7 GeV$^2$. In the high luminosity experiments, 89.3 pb$^{-1}$ data were accumulated. In total we have used data corresponding to 113.9 pb$^{-1}$ luminosity.

This thesis is organized as follows. We review the general formalism of the photon structure function and the theoretical predictions of the photon structure function,
in Chapter 2. In Chapter 3, we describe the TRISTAN accelerator and TOPAZ detector. The trigger system and the data acquisition system are also described here. In Chapter 4, we describe the performance of the TOPAZ Luminosity monitor, since Nara Women’s University High Energy Physics group is in charge of making this detector. In Chapter 5, we describe the methods of raw data processing. The event selection criteria used in this analysis is described in Chapter 6. In Chapter 7, we describe Monte Carlo simulations and analysis methods. In addition, we compare the Monte Carlo predictions to the observed data. In Chapter 8, we review the unfolding method developed by V. Blobel. The final results are discussed in Chapter 9. Conclusions are in Chapter 10.
Figure 2.1: Deep inelastic electron-photon collisions in two photon process.

Chapter 2

Theory of the Photon Structure Function
As mentioned in Chapter 1, the photon structure function can be studied using the reaction $e^+e^- \rightarrow e^+e^- + hadrons$ with the single-tag condition. In this Chapter, we first summarize the kinematics of the reaction and the formalism of the photon structure function. Next, several theoretical treatments of the photon structure function based on QPM and QCD are reviewed.

2.1 Formalism of the photon structure function

In the single-tag condition, the cross section of the process $e^+e^- \rightarrow e^+e^- X$ shown in Fig. 2.1 can be factorized into two parts. The cross section $(d\sigma_{e^+e^- \rightarrow e^+e^- X})$ is expressed as

$$d\sigma_{e^+e^- \rightarrow e^+e^- X} = d\sigma_{\gamma \rightarrow eX} \cdot f_{\gamma/e},$$  \hspace{1cm} (2.1)

where $d\sigma_{\gamma \rightarrow eX}$ is the cross section for deep inelastic electron-photon scattering and $f_{\gamma/e}$ is the target photon flux.

2.1.1 Deep inelastic electron-photon scattering

The term $d\sigma_{\gamma \rightarrow eX}$ in Eq. (2.1) corresponds to the cross section for deep inelastic electron-photon scattering depicted in Fig. 2.2.

The differential cross section for this process can be obtained as follows. If the
$+z$ coordinate is chosen along the incoming electron direction, the four momenta of each particle can be written as

\begin{align}
  p &= (E; 0, 0, E), \\
p' &= (E'; E' \sin \theta, 0, E' \cos \theta), \\
q &= p - p', \\
k &= (E_\gamma; 0, 0, -E_\gamma).
\end{align}

Here $p$ is the four momentum of the incoming electron with energy $E$ and $p'$ is that of outgoing electron with energy $E'$ and the scattering angle $\theta$. $k$ is the four momentum of the target photon and $q$ denotes the four momentum of the probing virtual photon. Using following definitions,

\begin{align}
  Q^2 &\equiv -q^2 > 0, \\
  \nu &\equiv q \cdot k,
\end{align}

and the mass of the hadron system $W$

\begin{equation}
  W^2 \equiv (k + q)^2.
\end{equation}

the scaling variables of $x$, $y$ and $z$ are defined by

\begin{align}
x &\equiv -\frac{q^2}{2\nu} = \frac{Q^2}{Q^2 + W^2}, \\
y &\equiv \frac{\nu}{p \cdot k} = 1 - \frac{E'}{E} \cos^2 \left(\frac{\theta}{2}\right), \\
z &\equiv \frac{\nu}{q \cdot p} = \frac{E_\gamma}{E}.
\end{align}

As shown in Fig. 2.2, the differential cross section for the deep inelastic scattering is expressed by using the amplitude associated with the electron-photon vertex, which is described by the leptonic tensor $L^{\mu\nu}$, and the amplitude associated with the virtual photon-real photon vertex, which is described by the hadronic tensor $W_{\mu\nu}$. The differential cross section in the laboratory frame is given by

\begin{equation}
  \frac{d^2 \sigma_{\gamma^*-eX}}{dE' d\Omega} = \frac{1}{16\pi^2} \frac{E'}{E} \frac{e^4}{Q^4} L^{\mu\nu} W_{\mu\nu},
\end{equation}

where the leptonic tensor $L^{\mu\nu}$ is well known in QED as follows:

\begin{equation}
  L^{\mu\nu} = 2 \left[ p^\mu (p')^\nu + p^\nu (p')^\mu - g^{\mu\nu}(p \cdot p' - m_e^2) \right],
\end{equation}
where the initial spin states of the electron are averaged and the final spin states are summed over. The hadronic tensor, $W_{\mu\nu}$, includes the effects of the structure in the real photon, which should be studied by experiments. The most general form of the hadronic tensor $W_{\mu\nu}$ can be obtained by requiring parity, time-reversal invariance and current conservations as follows:

$$ W_{\mu\nu} = W_1 \left( -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} \right) + W_2 \left( k_{\mu} - \frac{k \cdot q}{q^2} q_{\mu} \right) \left( k_{\nu} - \frac{k \cdot q}{q^2} q_{\nu} \right). \quad (2.14) $$

Here $W_1$ and $W_2$ are form factors to be determined by experiment. These form factors are only functions of the variables $\nu$ and $Q^2$ in general. The differential cross section for $e^+e^- \rightarrow e^+e^- + X$ is given by substituting Eq. (2.13) and Eq. (2.14) into Eq. (2.12) as follows:

$$ \frac{d^2 \sigma_{e^+e^- \rightarrow e^+e^- + X}}{d E' d\Omega'} = \frac{4\alpha^2 (E')^2}{Q^4} \left[ 2W_1(\nu, Q^2) \sin^2 \left( \frac{\theta}{2} \right) + W_2(\nu, Q^2) \cos^2 \left( \frac{\theta}{2} \right) \right], \quad (2.15) $$

where $\alpha$ is the coupling constant of QED.

Using $W_1$ and $W_2$, the photon structure functions is defined in a dimensionless form

$$ F_2^\gamma \equiv \nu W_2, \quad (2.16) $$

$$ F_1^\gamma \equiv W_1. \quad (2.17) $$

Using the scaling variables $x$ and $y$ instead of $\nu$ and $Q^2$, the differential cross section can be rewritten as follows:

$$ \frac{d^2 \sigma_{e^+e^- \rightarrow e^+e^- + X}}{d x dy} = \frac{16\pi \alpha^2 E E^\gamma}{Q^4} \left[ (1 - y) F_2^\gamma(x, Q^2) + xy^2 F_1^\gamma(x, Q^2) \right]. \quad (2.18) $$

In typical single-tagging experiments, the term $xy^2 F_1^\gamma(x, Q^2)$ can be neglected because $y$ is small ($xy^2 \sim 0.02$). Therefore one can measure the photon structure function $F_2^\gamma(x, Q^2)$ from the differential cross section in deep inelastic electron-photon scattering experiments.

### 2.1.2 Two photon process with single-tag condition

In actual experiments, deep inelastic electron-photon scattering is studied through two photon process, $e^+e^- \rightarrow e^+e^- + hadrons$ with the single-tag condition. As mentioned at the outset of Chapter 2, the reaction $e^+e^- \rightarrow e^+e^- + hadrons$ can be understood as two step processes. At first, the target real photon is radiated by the electron ( or positron ). The next step is the deep inelastic electron-photon
scattering. The differential cross section for the reaction $e^+e^- \rightarrow e^+e^-X$ can be written as follows:

$$\frac{d\sigma_{e^+e^- \rightarrow e^+e^-X}}{dxdy} = f_{\gamma/e}(z)dz \cdot \frac{d\sigma_{\gamma\gamma \rightarrow X}}{dxdy}, \tag{2.19}$$

where $\frac{d\sigma_{\gamma\gamma \rightarrow X}}{dxdy}$ is the differential cross section for the deep inelastic electron-photon cross section already mentioned in the Section 2.1. Assuming the target photon is a real photon, the factor $f_{\gamma/e}(z)$ means the target photon flux, which can be calculated by QED, as follows [9]:

$$f_{\gamma/e}(z) = \frac{\alpha}{\pi z} \left\{ \ln \frac{2E(1-z)}{m_e z} \left[ 1 + (1-z)^2 \right] - 1 + z \right\}, \tag{2.20}$$

where $z$ is the energy fraction of the target photon ($z = E_\gamma/E$) and $\theta_{\text{max}}$ is the maximum scattering angle of the electron emitting the target photon. This form is usually called Weizecker-Williams approximation.

From Eq. (2.18), (2.19) and (2.20), the photon structure function can be measured experimentally through the single-tag two photon process in $e^+e^-$collisions.

So far we have related the photon structure function $F_2^\gamma$ to experimental observables in $e^+e^- \rightarrow e^+e^-X$ reactions. In the next section, we review various theoretical calculations of the photon structure function. In these calculations $F_2^\gamma$ is derived from the cross section of virtual photon-real photon interactions ($\gamma^*\gamma \rightarrow \text{hadrons}$), as shown in Fig. 2.3.
2.2 The photon structure function in the Quark Parton Model

If the $Q^2$ of a virtual photon (probing one) is large enough, the photon interacts with the point-like constituents, called partons, inside the target photon. In the Quark Parton Model (QPM), quasi-free quarks are only treated as partons and gluon effects are ignored. In the QPM' picture, the scattering of the probing and target photon can be regarded as a sum of elastic scattering of the probing photon and a quasi-free quark inside a target photon. This approach leads to the identification of the photon structure function as a sum of the distribution of all quarks and antiquarks in the target photon as follows:

$$F_2(x, Q^2) = \sum_{i} n_f x \epsilon_i^2 \left[ q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right].$$  \hspace{1cm} (2.21)

Here $n_f$ is the number of quark flavors relevant in the reaction. The quark (antiquark) distribution $q_i(x, Q^2)$ ($\bar{q}_i(x, Q^2)$) describes the probability that the $i$th quark (antiquark) carries a momentum $xk$, where $k$ is the momentum of the target photon (see Fig. 2.4). The term $xc_i^2$ corresponds to the elementary cross section of the elastic scattering between a probing virtual photon $\gamma^*$ and a quark carrying an electric charge of $\epsilon_i$. In the QPM, the scaling variable $x$, which is defined in Eq. (2.9), is equal to the fraction of the photon momentum carried by the outgoing quark.
Figure 2.5: Feynman diagrams for the photon structure function in QPM.

In the framework of the QPM, the quark distribution in the photon can be obtained directly from the calculation of the cross section in the elementary process $\gamma^* \gamma \rightarrow q\bar{q}$.

For light quarks ($u, d$ and $s$), the quark distribution in the photon is given [10] by

$$q_i(x, Q^2) = 3 \alpha_i^2 \frac{\alpha}{2\pi} x \left\{ x^2 + (1 - x)^2 \ln \left( \frac{Q^2(1 - x)}{m_i^2 x} \right) + 8x(1 - x) - 1 \right\} . \quad (2.22)$$

This result is obtained from a calculation of box diagrams depicted in Fig. 2.5. Here $\alpha$ is the coupling constant of QED and $m_i$ is the mass of the quark with the electric charge $e_i$. The factor 3 comes from the number of quark colors. For antiquarks a similar form holds ($\bar{q}_i(x, Q^2) = q_i(x, Q^2)$). Using Eq. (2.21) and (2.22), the photon structure function is written as follows:

$$F_2(x, Q^2) = 3 \sum_{i=1}^{n_f} \alpha_i^2 \frac{\alpha}{2\pi} x \left\{ x^2 + (1 - x)^2 \ln \left( \frac{Q^2(1 - x)}{m_i^2 x} \right) + 8x(1 - x) - 1 \right\} . \quad (2.23)$$

The photon structure function has a $\ln Q^2$ dependence as seen in Eq. (2.23) in the QPM. This is one of the characteristics of the photon structure function. In the case of the proton structure function, $q(x)$ has no $Q^2$ dependence in the QPM level.

For heavy quarks ($c$ and $b$), a mass effect should be taken into account. The photon structure function for heavy quarks is given by

$$F_2(x, Q^2) = 3 \sum_{i=1}^{n_f} \alpha_i^2 \frac{\alpha}{2\pi} x \left\{ \Delta \left( 8x(1 - x) - 1 - \frac{4m_i^2}{Q^2} x(1 - x) \right) \right\}$$

$$\quad + \left\{ x^2 + (1 - x)^2 + 4 \frac{m_i^2}{Q^2} x(1 - 3x) - 8 \frac{m_i^4}{Q^4} x^2 \right\} \ln \frac{1 + \Delta}{1 - \Delta} \quad (2.24)$$
Figure 2.6: The corrections of gluon effects for $\gamma^*\gamma$ collisions. (a) shows the correction for the point-like component of the target photon, and (b) shows the correction for the hadronic component.

\[ \Delta^2 = 1 - \frac{4m_i^2x}{(1-x)Q^2}, \]

where $m_i$ is the mass of $i$th quark. In the low mass limit, Eq. (2.24) reduces to Eq. (2.23).

2.3 The photon structure function in Quantum Chromodynamics

Gluon effects in the photon structure function can be discussed using QCD. In the case that the photon has only point-like components, the diagrams of the QCD
corrections for the point-like component are shown in Fig. 2.6 (a). In addition to the point-like component of the photon, it is well known that photons behave like hadrons. The QCD correction for the hadronic component is shown in Fig. 2.6 (b), schematically. The QCD correction for the photon structure function becomes complicated because the photon structure function includes this hadronic component.

Historically there are two different methods in the calculation of the QCD correction to the photon structure function. One is the Operator Product Expansion (OPE) using the renormalization group equations [11][12][13]. This technique was used in the pioneering work of Witten [11]. The second approach uses the Altarelli-Parisi (AP) [14] evolution equation [15][16][17][18][19]. These two methods are confirmed to be equivalent in numerically [20]. Therefore, the basic formulae of the AP equation will be briefly sketched here because it is more transparent than the OPE technique.

### 2.3.1 The Altarelli-Parisi evolution equation for the photon structure function in leading log approximation

The parton distribution ($q_i$) in the photon is given by a sum of the quark distribution ($f_i$) and the gluon distribution ($f_g$) as follows:

\[
\frac{\partial q_i(x, \tau)}{\partial \tau} = \frac{\partial f_i(x, \tau)}{\partial \tau} + \frac{\partial f_g(x, \tau)}{\partial \tau},
\]

The AP evolution equations for the parton distributions in the photon [16] are given by

\[
\frac{\partial f_i(x, \tau)}{\partial \tau} = \frac{\alpha}{2\pi} P_{\gamma\rightarrow i\gamma}(x) + \frac{\alpha_s(\tau)}{2\pi} \int_x^1 \frac{dy}{y} \left\{ P_{\gamma\rightarrow j\gamma}(\frac{x}{y}) f_j(y, \tau) + P_{g\rightarrow j\gamma}(\frac{x}{y}) f_g(y, \tau) \right\},
\]

\[
\frac{\partial f_g(x, \tau)}{\partial \tau} = \frac{\alpha_s(\tau)}{2\pi} \int_x^1 \frac{dy}{y} \left\{ \sum_{i\gamma} P_{\gamma\rightarrow i\gamma}(\frac{x}{y}) f_i(y, \tau) + P_{g\rightarrow i\gamma}(\frac{x}{y}) f_g(y, \tau) \right\},
\]

where $P_{\gamma\rightarrow j\gamma}(x/y)$ are the leading order Altarelli-Parisi splitting functions, which give the probability that the particle $j$ carries a momentum fraction $x/y$ of the initial particle $i$. The explicit formulae of the splitting functions are summarized in Fig. 2.7. The variable $\tau$ is defined by

\[
\tau \equiv \ln \frac{Q^2}{\Lambda^2},
\]

where $\Lambda$ is the QCD scale parameter and $Q^2$ is the typical energy scale of the reaction. In the deep-inelastic $e\gamma$ scattering case, the square of the four momentum
Figure 2.7: The leading order Altarelli-Parisi splitting functions.
the probing photon \( Q^2 = -q^2 \) is taken as \( Q^2 \). The strong coupling constant \( \alpha_s \) is given in the leading logarithm order approximation as

\[
\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \ln \frac{Q^2}{\Lambda^2}},
\]

(2.29)

where \( n_f \) is the number of quark flavors.

The first term \( P_{\gamma \rightarrow q \bar{q}}(x) = 3[x^2 + (1 - x)^2] \) in Eq. (2.26) gives the probability that the photon splits into \( q \bar{q} \). This term corresponds to the QPM term mentioned before in Eq. (2.22). The existence of this first term is one of the most important features of the photon structure function. This term does not exist in the proton structure function. The other terms in Eq. (2.26) and Eq. (2.27) are the same as the evolution equations for the proton or neutron structure function.

If one writes the \( n \)-th moment of a function \( q \) as

\[
q^n = \int_0^1 x^{n-1} q(x) dx ,
\]

(2.30)

the solution of Eq.(2.25) in momentum space can be written as

\[
q^n_\gamma(Q^2) = \frac{4\pi}{\alpha_s(Q^2)} \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right)^{1 - d_\gamma} \right] A^n + \left( \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right)^{-d_\gamma} q^n_\gamma(Q_0^2) ,
\]

(2.31)

where the variables in the equation are defined as follows:

\[
d_\gamma^n = \frac{P_{\gamma \rightarrow q \bar{q}}}{2\pi(33 - 2n_f)}
\]

(2.32)

\[
A^n = \frac{a^n}{1 - d^n_\gamma}
\]

(2.33)

\[
a_n = \frac{\alpha}{4\pi} \frac{c^n_\gamma d^n_{\gamma \gamma}}{d^n_\gamma}
\]

(2.34)

\[
d^n_{\gamma \gamma} = \frac{P_{\gamma \rightarrow q \bar{q}}}{2\pi(33 - 2n_f)}.
\]

(2.35)

Here \( d^n_\gamma \), \( A^n \), \( a_n \) and \( d^n_{\gamma \gamma} \) are known functions determined from the splitting functions. The parton distribution \( q_\gamma \) therefore can be obtained from Eq. (2.31) for an input distribution \( q_\gamma(Q_0^2) \) at some scale \( Q_0^2 \). However this term can not be determined by perturbative QCD since the non-perturbative parts ( the terms \( \alpha_s(Q^2) > 1 \) ) are included in this term. Usually, either the prediction of the Vector Meson Dominance (VMD) model or the experimental data are used to define the input distribution \( q_\gamma(Q_0^2) \). For example, the parameterization for the input distribution using experimental data has been undertaken by Drees and Grassie (DG) [18] and by Abramowicz, Charchula and Levy (LAC) [19]. This approach is identical to the approach applied to the proton ( or neutron ) case. In the next subsection these parametrizations will be reviewed in more detail.
2.3.2 The parametrizations of parton distribution in the photon

DG parameterization

The approach using the AP evolution of the photon structure function has been undertaken [18] by Drees and Grassie. They used the AP equation to evolve an input parametrization at $Q = Q_0$. The PLUTO data at $Q^2 = 5.9\text{GeV}^2$ were used as the input distribution of $q_g(Q_0^2)$ at $Q_0^2 = 1\text{GeV}^2$. The input distribution is started with 3 flavors and evolved to higher $Q^2$ with 4 and 5 flavors. For the QCD scale parameter $\Lambda$, they assumed a value of 0.4 GeV. Although the gluon distribution in the photon is included, the data available were not precise enough to determine the gluon distribution unambiguously. Fig. 2.8 shows the DG parametrization of the photon structure function $F_2^{DG}$ at $Q^2 = 5.9\text{ GeV}^2$ using PLUTO data.

LAC parameterization

The same procedure as DG has recently been applied to all existing experimental data on the photon structure function by Abramowicz, Charchula and Levy (LAC)
Figure 2.9: A comparison of the prediction of the LAC parametrization with $Q_0^2 = 1 \text{GeV}^2$ (dashed line), with $Q_0^2 = 4 \text{GeV}^2$ (full line) and the DG parametrization (dotted line) with the measurement of $F_2^* \gamma$ as a function of $x$ for different values of $Q^2$.

[19] They used the evolution equation with four flavors to fit the input distribution. They assumed a QCD scale parameter $\Lambda = 0.2 \text{ GeV}$. The parametrization is provided for different starting values of $Q_0^2$, ranging from 1 to 4 $\text{GeV}^2$. Above 2 $\text{GeV}^2$, the results do not depend on the initial conditions. The data are well reproduced for all starting values.

Fig. 2.9 shows a comparison of the prediction of the LAC parametrization and the DG parametrization with previous experimental data taken at $e^+e^-$ machines: PETRA, PEP and TRISTAN (AMY). In general, above $Q^2 = 5 \text{ GeV}^2$, both the DG and the LAC parametrization reproduce the experimental data quite well. Below $Q^2 = 5 \text{GeV}^2$, the LAC parametrization with $Q_0^2 = 1 \text{GeV}^2$ gives a better description of the experimental data. The basic difference between the DG parametrization and the LAC with $Q_0^2 = 4 \text{GeV}^2$ is seen in the very low $x$ region, where the LAC
Figure 2.10: Schematic diagram of the photon structure function in QCD.

prediction shows a sharp rise of the structure function.

2.3.3 FKP formula

Another approach to the QCD calculation for the photon structure was carried out by Field, Kapusta and Pogioli (FKP) [21] [22]. They introduced a single phenomenological parameter, $p_t^2$ or $t_c$, in order to separate the “hadronic-part” ( = non-perturbative part ) and the point-like part ( = perturbative part ) of the photon structure function. Here $p_t$ is the boundary of the transverse momentum ( $p_t$ ) of the produced quark at the target photon vertex, and $t_c$ is the boundary of the square of four momentum transfer ( $t$ ) of the virtual quark at the target photon vertex ( see Fig. 2.10 ). This approach is formally equivalent to the approach by introducing a finite input distribution at $Q_0^2$ in the AP equation.

The hadronic and point-like parts of the photon structure function are separated into two parts using the cut-off of the momentum transfer $t_c$ as follows:

$$F_2 = \int_{t_{min}}^{t_c} \frac{dF_2^{hadronic}}{dt} dt + \int_{t_c}^{t_{max}} \frac{dF_2^{point-like}}{dt} dt ,$$  \hspace{1cm} (2.36)
where $t$ is kinematically related to the transverse momentum ($p_t$) of the quark as follows:

$$t = \frac{Q^2}{2x} \left(1 \pm \sqrt{1 - \frac{4(p_t^2 + m_q^2)}{W^2}}\right).$$  \hfill (2.37)

The point-like (perturbative) part was calculated by Kapusta [22] using AP splitting functions. The solution is as

$$I^Q_{CD} = \frac{3\alpha}{\pi} \sum_i e_i^4 x \left[6x(1-x) + \frac{2m_i^2}{t_c} x - 1 + \frac{a(x)}{x^C + C f(x)} \right] Y_{\max} \left\{1 - \left(\frac{Y_c}{Y_{\max}}\right)^{1+C f(x)}\right\},$$  \hfill (2.38)

$$f(x) = 2\ln \frac{1}{1-x} - x - \frac{1}{2} x^2,$$  \hfill (2.39)

$$c = \frac{33 - 2n_f}{8},$$  \hfill (2.40)

$$Y_{\max} = \ln \left(\frac{t_{\max}}{\Lambda^2}\right) = \ln \left(\frac{Q^2}{x \Lambda^2}\right),$$  \hfill (2.41)

$$Y_c = \ln \left(\frac{t_c}{\Lambda^2}\right),$$  \hfill (2.42)

$$t_c = \frac{m_i^2 + (p_t^0)^2}{1-x}.$$  \hfill (2.43)

The equation has two free parameters, one is the QCD scale parameter $\Lambda$ and the other is a cut-off parameter either $p_t^0$ or $t_c$. To show the sensitivity of $\Lambda$ and $p_t^0$ to the photon structure function, $F_{2\gamma}^\gamma$ is shown with different values of $\Lambda$ and $p_t^0$ in Fig. 2.11. In figure (a), $p_t^0$ is changed from 0.1 to 0.9 GeV/c with the value of $\Lambda = 0.2$ GeV, and in the figure (b), $\Lambda$ is changed from 0.1 to 0.9 GeV with the value of $p_t^0 = 0.5$ GeV/c. The photon structure function is insensitive to the value of $\Lambda$ but is very sensitive to changes in $p_t^0$. In this thesis, we have determined the value of $p_t^0$ using experimental data. The statics of our data were not enough to determine $\Lambda$ with how precision.

In the FKP approach, the VMD formula $F_{2\gamma}^\gamma = 0.2\alpha(1-x)$ is taken to estimate the hadronic (non-perturbative) part of the photon structure function.
Figure 2.11: The photon structure function in FKP formula. (a) show the dependence of $p_t$ for $F_2^{FKP}$ at the QCD scale parameter $\Lambda = 0.2$ GeV and (b) shows that of the $\Lambda$ at $p_t = 0.5$ GeV.
Chapter 3

Experimental Apparatus

3.1 TRISTAN accelerators

TRISTAN is the $e^+e^-$-accelerator-collider at Japan’s National Laboratory for High Energy Physics (KEK). As shown in Fig. 3.1, the collider consists of three different accelerators; a 400m linear accelerator (LINAC), an accumulator ring (AR) 377m in circumference and a main ring (MR) about 3km in circumference. Electrons and positrons are accelerated up to 2.5 GeV by the LINAC and injected into the AR. There they are accelerated up to 8 GeV and injected to the MR with a current sufficient for experiment. Two bunches of electrons and positrons are stored in the ring and accelerated up to $\sim 30$ GeV and then made to collide every 5 $\mu$sec. The center of mass energy of TRISTAN was initially 50 GeV in autumn, 1986, and was gradually raised to the highest energy, 64 GeV, at the end of 1989. In this period the maximum daily integrated luminosity was $350 \text{ nb}^{-1}/\text{day}$. In spring, 1990, the operation mode of TRISTAN was changed from higher energy runs to higher luminosity runs. The center of mass energy has been fixed at 58 GeV for high luminosity data taking. As shown in Fig. 3.2, 700 $\text{nb}^{-1}/\text{day}$ data at the maximum, were taken in high luminosity runs. The specific luminosity $(L/I^+I^-)$ during higher energy runs was $\sim 0.3$ and during higher luminosity runs was $\sim 0.7$ as shown in Fig. 3.3.

3.2 TOPAZ detector

The TOPAZ detector [23] is a general purpose detector at TSUKUBA experimental hall in the north east part of the TRISTAN main ring. Main feature of the TOPAZ detector is a Time of Projection Chamber as the central tracking device.
Figure 3.1: A plain view of TRISTAN.
Figure 3.2: Integrated luminosity per day.

Figure 3.3: Specific luminosity. (a) shows at higher energy runs and (b) at higher luminosity runs.
The bird’s eye and the cross-sectional view of the detector are shown in Fig. 3.4 and Fig. 3.5, respectively. For detecting charged particles, an Inner Drift Chamber (IDC) and the Time Projection Chamber (TPC) were installed. For measuring the energy of electrons and photons, an End-cap Calorimeter (ECL) and a Barrel Calorimeter (BCL) were installed. In order to measure the beam luminosity, a Luminosity Monitor (LUM) was installed at small polar angle.

The Luminosity monitor (LUM) and the Inner Drift Chamber (IDC) were later replaced by a Forward Calorimeter (FCL), a Vertex Chamber (VTX) and a Trigger Chamber (TCH) in autumn 1989, 1990 and 1989, respectively. These new detectors will be described in greater detail in Section 3.3.

The performance of the TOPAZ detector is summarized in Table 3.1. The TOPAZ detector coordinate system used in this thesis is shown in Fig. 3.6.

### 3.2.1 Inner Drift Chamber (IDC)

The Inner Drift Chamber [24] was a cylindrical wire drift chamber, which was the innermost element of the TOPAZ detector. It was used as a fast triggering element for charged particles (charged pre-trigger) and for tracking of charged particles to assist the Time Projection Chamber tracking. The inner and outer radii, and the
Figure 3.5: A cross-sectional view of the TOPAZ detector

Figure 3.6: A coordinate system of the TOPAZ detector.
<table>
<thead>
<tr>
<th>Components</th>
<th>Material and Size</th>
<th>Measured Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam pipe</td>
<td>0.2m(\phi)</td>
<td>( \sigma_{r \phi} = 220\mu m ) &lt;br&gt;1.8 mm-thick Al</td>
</tr>
<tr>
<td>IDC</td>
<td>Cylindrical</td>
<td>( \sigma_{r \phi} = 220\mu m ) &lt;br&gt;10 layers</td>
</tr>
<tr>
<td>TPC</td>
<td>2.4m(\phi) \times 2.2m(L)</td>
<td>( \sigma_{r \phi} = 230\mu m ) &lt;br&gt;16 sectors</td>
</tr>
<tr>
<td>TOF</td>
<td>Plastic scinti.</td>
<td>( \sigma_{time} = 220 \text{ ps (Bhabha)} )</td>
</tr>
<tr>
<td>Magnet</td>
<td>2.9m(\phi) \times 5m (L), 0.7X_0</td>
<td>( \sigma_{r \phi} = 350\mu m )</td>
</tr>
<tr>
<td>BDC</td>
<td>Streamer tube</td>
<td>( \sigma_{r \phi} = 350\mu m )</td>
</tr>
<tr>
<td>BCL</td>
<td>Lead-glass (SF6W), 20X_0 thick</td>
<td>( \sigma_{E/E} = \sqrt{(8.0/\sqrt{E})^2 + (2.5)^2}% )</td>
</tr>
<tr>
<td>BMU</td>
<td>3 layers(( r \phi )), 1 layer(z)</td>
<td>( \sigma_{r \phi} = 1\text{mm} )</td>
</tr>
<tr>
<td>LUM</td>
<td>Pb-Scinti. Sandwich</td>
<td>( \sigma_{E/E} = 3.5% ) &lt;br&gt;38 mrad &lt; ( \theta &lt; 66 \text{ mrad} )</td>
</tr>
<tr>
<td>EDC</td>
<td>Streamer tube</td>
<td>( \sigma_{E/E} = 6.7% )</td>
</tr>
<tr>
<td>ECL</td>
<td>Pb-PWC Sandwich</td>
<td>( \sigma_{E/E} = 6.7% ) &lt;br&gt;18X_0 thick</td>
</tr>
</tbody>
</table>

Table 3.1: The performance of the TOPAZ Detector
length along the beam line of the Inner Drift Chamber were 11 cm, 29 cm, and 1.6 m, respectively. The Inner Drift Chamber had 10 layers of anode wires and 8 layers of cathode delay lines to measure position in the z-direction. Its cylinders were filled with a gas mixture of $Ar(89\%) + CO_2(10\%) + CH_4(1\%)$, called HRS gas. The spatial resolutions were $\sigma_{r\phi} = 220\mu m$ and $\sigma_z = 1cm$, respectively.

3.2.2 Time Projection Chamber (TPC)

The Time Projection Chamber [25] is the central tracking device of the TOPAZ detector. It measures precise 3-dimensional positions and ionization energy losses of charged particles for particle identifications.

The Time Projection Chamber is located outside the Inner Drift Chamber, inside a cylindrical pressure vessel of 0.3m $\sim$ 1.29m in radius and 2.6m in length along the beam axis. The cylindrical pressure vessel is filled with gas mixture of $Ar(90\%) + CH_4(10\%)$ at 3.5 atm. The structure of the Time Projection Chamber is shown in Fig. 3.7.

Passing through the chamber gas volume, the charged particle ionizes the gas
molecules. These ionization electrons are then drifted to the endcaps with a speed of 5.3 cm/μsec along the $z$ direction by an electric field of 35 KV/m. To produce the field, a negative wire voltage is applied to a central membrane in the center of the pressure vessel. Since a magnetic field of 1 Tesla is applied parallel to the electric field, the diffusion of the electrons is reduced by cyclotron motion. The electrons finally reach the endcaps of the cylinder and are detected by one of 8 multi-wire proportional counters (sector). Each sector has 176 sense wires and 512 segmented cathode pads, as shown in Fig. 3.8. Electrons drifted into a sector cause an electron avalanche near the sense wires and ion clouds induce an image charge on the cathode pads. The signals from wires and pads are amplified and shaped by analog electronics. Then signals go into Charged Coupled Device (CCD) digitizers and are sampled and stored in units of 100 nsec “buckets”.

The $(r - \phi)$ position of a particle is calculated from the signals of the cathode

Figure 3.8: A sector of the Time Projection chamber.
Figure 3.9: The Gating grid of the Time Projection Chamber. (a) shows the open mode and (b) shows the shut mode.

pads. The $z$ position is determined from the arrival time of electrons relative to the triggered time which is defined by a Beam Crossover (BCO) signal. The energy loss measurements are obtained from pulse height of sense wire signals.

Positive ions, moving $10^4$ times slower than electrons, built up into a positive ion cloud in the sensitive region of the TPC. This effect can cause a distortion in track reconstruction, of as large as a few cm. To avoid such effects, a gating grid system [26], shown in Fig. 3.9, was installed. Usually the gating grid is closed so that positive ions can not flow into the sensitive volume. Only when opening signals come from other detectors, will the gating grid open. We will describe how the gating grid opens in a later section.

A typical position resolution of the TPC is $\sigma_{r\phi} = 185\pm 4 \mu \text{m}$ and $\sigma_z = 335\pm 2 \mu \text{m}$. The $dE/dX$ resolution is $\sigma_{dE/dX} = 4.6 \pm 0.1\%$. The momentum resolution as a function of momentum is shown in Fig. 3.10 and is obtained to be: $d\sigma/P_t = \sqrt{(1.5P_t)^2 + (1.6)^2\%}.$
Figure 3.10: Momentum resolution of the Time Projection Chamber as a function of $P_t$. 
3.2.3 Time of Flight counter (TOF)

The Time of Flight Counters [27] are cylindrically placed outside the time projection chamber with inner and outer radii of 1.29m and 1.36m, respectively. The detector consists of plastic scintillators segmented into 64 parts in the $\phi$ direction and measures the time of flight of charged particles. Its typical time resolution is 220 psec. It is also used for the charged pre-trigger together with the inner drift chamber.

3.2.4 Superconducting solenoid magnet

A thin superconducting solenoid magnet [28] is located outside the Time of Flight Counter. It provides a uniform magnetic field of 1 Tesla parallel to the beam line for momentum measurement in the Time Projection Chamber and Inner Drift Chamber. The total radiation length of the magnet is 0.73 $X_0$.

3.2.5 Barrel Calorimeter (BCL)

The Barrel electromagnetic Calorimeter [29] is placed between the superconducting solenoid and the return yoke of the magnet. It measures the energies of electrons, positrons and photons precisely.

A schematic view of the Barrel Calorimeter is shown in Fig. 3.11. The Barrel Calorimeter has a cylindrical shape with an inner radius of 1.76 m and a length of 5.6m along the beam line. It covers 85% of the full solid angle corresponding to $32^\circ \leq \theta \leq 148^\circ$, where $\theta$ is the polar angle measured from the beam axis.

The BCL consists of 4300 blocks of lead glass Cherenkov counters. A counter block is made of SF6W lead glass. One block is $112\text{mm} \times 113\text{mm}$ at the front and $112\text{mm} \times 135\text{mm}$ at the end of the block, and the radiation length of each block is $20X_0$ so as to absorb more than 95% of the shower energies at $\sim 30$ GeV. The lead glass blocks are tilted by $1.8^\circ$ with respect to the radial line in the $(r - \phi)$ plane so that electrons and photons cannot escape by passing through counter to counter gaps.

Fig. 3.12 shows the energy distribution for Bhabha events at $\sqrt{s} = 52$ GeV. The energy resolution of the BCL is 4.5% and the angular resolution is $\sigma_\theta = 0.38^\circ$. 
Figure 3.11: The schematic view of the Barrel Calorimeter.

Figure 3.12: The energy distribution for Bhabha events with the Barrel Calorimeter.
3.2.6 Barrel Muon Drift Chamber (MDC)

The Barrel Muon Drift Chamber is the outermost element of the TOPAZ detector. It consists of three layers of drift chambers interleaved with iron filters. The Muon Drift Chamber identifies muons which pass through the iron filters while hadrons are absorbed in the filters. The tracking efficiency is $89.5 \pm 0.3\%$ for cosmic ray events. Its spatial $(r-\phi)$ resolution is $\sigma_{r,\phi} = 1 \text{mm}$.

3.2.7 Luminosity Monitor (LUM)

The Luminosity Monitor [30] worked very well from May 1987 to autumn 1989. In order to detect small angle Bhabha scattering the Luminosity Monitor was located at a small polar angle. Main roles of the Luminosity Monitor were to measure the real time beam luminosity as the online monitor, to know the specific luminosity (the ratio of the luminosity to the product of electron and positron currents) for each beam fill, and to measure the integrated luminosity for each day or experiment.

The Luminosity Monitor consisted of four sets of counters as shown in Fig. 3.13. Each set of counters (Fig. 3.14) consists of a Defining (D), a Complementary (C), and Shower (S) counter. The Defining counter and the Complementary counter were made of plastic scintillators. The Shower counter was made of 36 alternating layers of lead and scintillator with a thickness of 2.8mm and 5.0mm, respectively. The
radiation length of the Shower counter is 18.5. Light signals from the scintillator were transferred via a 2.0 m plastic optical fiber to a photomultiplier located outside the magnetic field. The angle subtended by a requiring a hit in a Defining counter and a Complimentary counter on opposite side of the interaction point (for example $D_1$ and $C_4$) defines the acceptance for Bhabha events. The polar angular coverage of the Defining counter for Bhabha events is from 38 to 66 mrad. To cover the size of beam interaction the Complimentary counter is made larger than the Defining counter.

Fig. 3.15 shows the energy distribution for Bhabha events. The energy resolution for Bhabha events at the beam energy 26 GeV is $\sigma_E/E = 3.5\%$.

3.2.8 Endcap Drift Chamber (EDC)

The Endcap Drift Chamber [31] is located in front of the Endcap Calorimeter. The Endcap Drift Chamber consists of 4 layers of drift chambers made of conductive plastic tubes filled with a gas mixture of $Ar(20\%) + C_2H_6(80\%)$ at 1 atm and is operated in the limited streamer mode. The main role of the Endcap Drift Chamber is to improve momentum resolution in the endcap region outside of the acceptance region of the Time Projection Chamber.
Figure 3.15: The energy distribution for Bhabha events with the Luminosity Monitor.
3.2.9 Endcap Calorimeter (ECL)

The Endcap Calorimeter [32] is a gas sampling calorimeter covering the angular region $0.85 \leq |\cos \theta| \leq 0.98$. It consists of 34 layers proportional counters sandwiched between lead absorber of 2 or 3mm thickness. One layer consists of 14 proportional tubes made of conductive plastic of $15 \text{mm} \times 10 \text{mm}$ cross section. As Fig. 3.16 shows, the energy resolution of the Endcap Calorimeter for Bhabha events at $\sqrt{s} = 52$ GeV is 6.7%. The angular resolution is $\sigma_\theta = 0.7 \text{deg}$.

3.3 New detectors

For high luminosity experiments, the TOPAZ detector was upgraded in autumn 1989. Fig. 3.17 shows a cross-sectional view of the upgraded TOPAZ detector. In calorimetry, the Forward Calorimeter (FCL) and a Ring Calorimeter (RCL) have made the TOPAZ detector hermetic together with the Barrel Calorimeter and the Endcap Calorimeter [33]. The Inner Drift Chamber was replaced by two kinds of tracking chambers; the Trigger Chamber (TCH) and the Vertex Chamber (VTX), in 1989 and 1990, respectively [34]. A new Forward Muon Chamber (F/B MDC)
Figure 3.17: A cross-sectional view of the upgraded TOPAZ detector
was supplemented and extended muon identification capability. Performance of new detectors is summarized in Table 3.2.

### 3.3.1 Vertex Chamber (VTX)

The Vertex Chamber [35] is a high precision tracking chamber at the center of the TOPAZ detector. Fig. 3.18 shows a sketch of mechanical substructure of the Vertex Chamber. It consists of a beryllium beam pipe as the inner wall of the chamber, two ceramic endplates, an outer carbon-fiber-epoxy cylinder to support the endplates and finally a pressure vessel.

The chamber is divided into 16 sectors with 25 sense wires. The wire configuration is shown in Fig. 3.19. The sectors are tilted 16° relative to the radial direction to resolve the left-right ambiguity and to achieve good tracking acceptance. The pressure vessel is filled with the slow gas ($CO_2(92\%) + C_2H_6(8\%)$) pressurized to 3 atoms.

The local spatial resolution and the detection efficiency were studied with cosmic ray events and 29 GeV Bhabha events, to be $\sigma_{r_\phi} = 30 \sim 50 \mu m$ and greater than 99%, respectively.

<table>
<thead>
<tr>
<th>Components</th>
<th>Material and Size</th>
<th>Measured Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>VTX</td>
<td>Jet-type</td>
<td>$\sigma_{r_\phi} = 30 \sim 50 \mu m$</td>
</tr>
<tr>
<td></td>
<td>using slow gas $CO_2/C_2H_6(92/8)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25 sense wires/sector $\times$ 16 sectors</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$66 \text{ mm} \leq r \leq 160.5 \text{ mm}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max drift length : 32 mm</td>
<td></td>
</tr>
<tr>
<td>TCH</td>
<td>Cylindrical</td>
<td>$\sigma_{r_\phi} = 250 \sim 300 \mu m$</td>
</tr>
<tr>
<td></td>
<td>HRS gas</td>
<td></td>
</tr>
<tr>
<td></td>
<td>128 sense wires/layer $\times$ 8 layers</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$20 \text{ cm} \leq r \leq 30 \text{ cm}$</td>
<td></td>
</tr>
<tr>
<td>EMU</td>
<td>2 layers (x), 2 layers (y)</td>
<td>$\sigma_{r_\phi} = 1 \text{ cm}$</td>
</tr>
<tr>
<td></td>
<td>Coverage $0.91 \times 4\pi$ (BMU+EMU)</td>
<td></td>
</tr>
<tr>
<td>FCL</td>
<td>BGO + Si-strip, 21$X_0$ thick</td>
<td>$\sigma_E/E \approx 5.0%$ (for 28 GeV electron)</td>
</tr>
<tr>
<td></td>
<td>$60\text{mrad} &lt; \theta &lt; 188\text{mrad}$</td>
<td></td>
</tr>
<tr>
<td>RCL</td>
<td>W-Si Sandwich, 6.7$X_0$ thick</td>
<td>$\sigma_E/E \approx 20%$ (for 28 GeV electron)</td>
</tr>
<tr>
<td></td>
<td>$0.82 \leq</td>
<td>\cos \theta</td>
</tr>
</tbody>
</table>

Table 3.2: The performance of new detectors
Figure 3.18: A sketch of mechanical substructure of the Vertex Chamber.

Figure 3.19: The wire configuration of the Vertex Chamber.
3.3.2 Trigger Chamber (TCH)

The Trigger Chamber is a cell type cylindrical wire chamber placed between the Vertex Chamber and the Time Projection Chamber. The inner and outer radii and the length of the Trigger Chamber are 20cm, 30cm and 130cm, respectively. The major purposes of this chamber are to provide a fast trigger signal in place of the Inner Drift Chamber and to give additional information for track linking between the Vertex Chamber and the Time Projection Chamber. The Trigger Chamber consists of eight layers of sense wires which are azimuthally segmented into 128 cells per layer. The shape of each cell is hexagonal and the size of the cell is about 1cm. The chamber is filled with the HRS gas \((\text{Ar}(89\%) + \text{CO}_2(10\%) + \text{CH}_4(1\%))\) at 1 atm. The spatial resolution of the Trigger Chamber is \(\sigma_{r\phi} = 250 \sim 300\mu\text{m}\) for cosmic ray events. The chamber efficiency and the trigger efficiency are 98% and 99.8%, respectively.

3.3.3 Forward Calorimeter (FCL)

The forward calorimeter [36] is located at a small polar angular region of the TOPAZ detector. It was installed in autumn 1989 to replace of the Luminosity Monitor. It’s main purposes are to make the TOPAZ detector hermetic, to tag electrons or positrons in order to study two-photon processes in detail, to monitor real-time luminosity and to measure the integrated luminosity.
Figure 3.21: The cross-sectional view of Forward Calorimeter.
Figure 3.22: The structure of BGO Calorimeter.

As Fig. 3.20 shows, it comprises four identical modules. We call the two modules at \( z = \pm 60\text{cm} \) the Forward-Backward Calorimeter (FBC) and the other two modules at \( z = \pm 120\text{cm} \) the Luminosity Monitor (LUM) for convenience. The modules cover polar angles from 3.6° to 13.6° and the full azimuthal angle. Each module is azimuthally segmented into 12 blocks, as can be seen in Fig. 3.21. Each block consists of a Si-strip detector and a BGO calorimeter.

**BGO calorimeter** As shown in Fig. 3.22, a BGO calorimeter consists of a 5.3 mm thick tungsten absorber (1.5\(X_0\)) and a 150 mm thick BGO crystal (13.5\(X_0\)). Each block consists of a Si-strip detector and a BGO calorimeter. The tungsten absorber is used to reduce the longitudinal shower leakage from the calorimeter and to keep the energy resolution less than 5% from 1 to 30 GeV. Fig. 3.23 shows the energy
resolution of the BGO calorimeter as a function of incident beam energy. The solid squares are the results of test beam measurements. The cross point was obtained from Bhabha events at $\sqrt{s} = 58$ GeV. The open circles represent the Monte Carlo predictions for a thickness of $1.5X_0$ (W) and $13.5X_0$(BGO) based on EGS4 [37]. The resolution at lower energy($\leq 3$GeV) is determined by an absorber(W) effect, and that at higher energy($\geq 3$GeV) is determined by a longitudinal shower leakage effect. We can see in Fig. 3.23 the energy resolution is less than 5% for $\geq 3$ GeV. Fig. 3.24 compares the deposit energy distribution for Bhabha events at $\sqrt{s}$ with the Monte Carlo simulation.

Si-strip detector  A Si-strip detector is placed in front of the BGO calorimeter. A schematic view of the Si-strip detector is shown in Fig. 3.25. The p-side comprises 12 strips, and the n-side is used as the common ground. The width of the strips ranges from 5 to 8 mm, so as to keep the angular resolution ($\Delta \theta/\theta$) at less than 5%. Fig. 3.26 shows the minimum ionizing peak of Si-strip detector for 29 GeV Bhabha events with electrical noise samples. The signal and noise are clearly separated, and the signal to noise ratio (S/N) is about 16. We can see good uniformity of
Figure 3.24: Energy distribution for Bhabha events with the Forward Calorimeter.
Figure 3.25: Structure of the Si-strip detector.
Figure 3.26: Pulse height distribution of the Si-strip detector.
the azimuthal-angle distribution for Bhabha events measured by LUM and FBC in Fig. 3.27.

3.3.4 Ring Calorimeter (RCL)

The Ring Calorimeter is located between the end cap and the Barrel Calorimeter in order to fill the gaps of these calorimeters. The main purpose of the RCL is to veto photons and electrons. The RCL consists of two rings. Each ring is made of two-layers of 300μm Si with a respectively heavy metal \((W(95\%) + Ni/Cu(5\%))\), whose radiation lengths are \(5.7X_0\) and \(1.3X_0\), respectively. In the \(\phi\) direction, each layer is segmented with 96 Si-W counters, which consists of 6 Si-pads. The inner and outer radii of each counter is 96 cm and 108 cm, respectively. The energy resolution
for Bhabha events at $\sqrt{s} = 58$ GeV was $\sigma_E/E = 17.5 \pm 2.1\%$.

### 3.3.5 Forward-backward Muon Chamber (F/B MU)

The Forward-backward muon chamber covers a solid angle region $0.66 < | \cos \theta | < 0.89$. It consists of a 30 cm iron absorber and a total of 4 drift planes.

### 3.4 Trigger system

The purpose of the trigger system is to pick interesting events from huge backgrounds effectively. The time taken for the acquisition of one accepted events is about 30 msec. This period is called “dead time” because other trigger signals cannot be accepted. In order to keep the dead time less than 10%, the total trigger rate should be less than 3 Hz.

Efficient data taking is also needed in order to reduce the raw data size. Our raw data size is very large; for example, 150 Kbytes for multi-hadron events and 50 Kbytes for most triggered events. This is due to the data size of the Time Projection Chamber which acquires many kinds of information. It is very important for the trigger system to select significant events.

To reduce the dead time and the data size, the trigger system must not only suppress background such as beam-gas events, spent electrons and cosmic ray events, but must also select interesting events with high efficiency.

The TOPAZ trigger system [38] consists of two hardware parts shown in Fig. 3.28 and one software [39] part which has been used for high luminosity experiments since February, 1990. One part of the hardware trigger uses energy deposit information, called the “Energy Trigger”. The other part is a trigger using charged tracks from devices, such as Inner Drift Chamber or Trigger Chamber, Time Projection Chamber, and Time of Flight Counter, called the “Charged Trigger”. In the software trigger, a more accurate decision is made by a multi-processor system using timing information from the Time Projection Chamber, called the “Software Trigger”. In the next sections, we describe the Energy Trigger, the Charged Trigger and the Software Trigger.

### 3.4.1 Energy Trigger

A block diagram of the Energy Trigger is shown in Fig. 3.29. Bhabha events,
Figure 3.28: A block diagram of the TOPAZ trigger system.
Figure 3.29: A block diagram of the Energy Trigger.
hadronic events and two-photon events are triggered mainly by the Energy Trigger. An Energy Trigger signal is generated by OR’s of following conditions.

1. Total deposit energy is greater than $4$ GeV in the Barrel Calorimeter.

2. There are at least two energy deposits greater than $1$ GeV among the forward, center and backward region of the Barrel Calorimeter.

3. Total deposit energy is greater than $10$ GeV in the Endcap Calorimeter.

4. There are two energy deposits greater than $3$ GeV, in both the forward and backward Endcap Calorimeter.

The typical rate of the Energy Trigger is about $1.5$ Hz.

### 3.4.2 Charged Trigger

The charged Trigger has two decision levels. The first level decision is called “Pre-trigger” and the second is “TPC Trigger”. The Pre-trigger is used to open the TPC gating grid. The TPC Trigger starts when the Pre-trigger is generated. On generating the TPC Trigger, an on-line micro-processor starts data taking. This trigger system is described below.

**Pre-trigger** The Pre-trigger mainly consists of “Forced Pre-trigger” and “Charged Pre-trigger.” The Forced Pre-trigger is generated by the Energy Trigger. In addition, random trigger ($\sim 0.1$ Hz) and scaled down LUM trigger are added to the Forced Pre-trigger. The Charged Pre-trigger is made by using the signals of the Time of Flight counter and the Inner Drift Chamber. Two or more track segments should be found in the Inner Drift Chamber (or Trigger Chamber for higher luminosity runs) corresponding to hits in the Time of Flight counter. The rate of this trigger is a few hundred Hz.

**TPC Trigger** The TPC Trigger consists of a Ripple Trigger, a Vertex Finder and a LAM controller [40]. The Ripple Trigger finds tracks coming from approximately inside the detector. For the tracks found by the Ripple Trigger, the vertex position is calculated in the Vertex Finder. Finally, the LAM controller makes the trigger decision by using the information from the Forced Pre-trigger and the track pattern in TPC.
Each TPC sector has 176 sense wires. From these wires, 88 wires close to the beam line are used for the TPC trigger. In each TPC sector, eight adjacent wires are regarded as one wire-group (WG) and eleven wire-groups are made up as shown in Fig. 3.30. The length of each wire group is 3.2 cm in the radial direction. The edge of the WG1 in the Fig. 3.30 is located at 372 mm from the beam line. In order to remove noise hits, these eight wires are divided into two groups of four alternative wires in each wire group. The outputs of four alternative wires are ORed and two ORed outputs are ANDed, as shown at the bottom of Fig. 3.30.

Using the eleven wire-group outputs, tracks from the event vertex are searched for. Fig. 3.31 shows an example of a track originating from the beam collision. The signal of WG11 is the fastest signal and WG10 is slightly delayed from WG11. So there should be an overlapping of the signal WG11 and WG10, WG10 and WG9, and so on. The time difference between the leading edge of the adjacent wire group signal is 1\(\mu\) sec. The trailing edge of each signal is stretched by a one-shot of 1\(\mu\)sec. So that the stretched wire group signal has approximately 1.5\(\mu\)sec width ( 0.5 \(\mu\)sec from the shaping amplifier constant plus 1\(\mu\) sec one-shot ). After this, the Ripple Trigger is used to detect this signal continuity. If a track generates eleven continuous wire group hits, the Ripple Trigger recognizes it as a track. The efficiency of the logic is estimated to be more than 98.5% using cosmic ray muons.

For clear track candidates in the Vertex Finder, the vertex point is calculated using two points W1 (WG1) and W2 (WG6) as shown in Fig. 3.31. The vertex finder requires the vertex point of the tracks to be within \(\pm 20\) cm as shown in Fig. 3.32.

After the tracks from the vertex are selected sector by sector, a final trigger decision is made by the LAM Controller. At first, if there is a Forced Pre-trigger, the TPC Trigger is issued without looking at the TPC decision. Otherwise if the TPC’s hit-pattern, such as single hit, back to back, more than \(n\) tracks, etc., matches with the mask set, the TPC Trigger is issued. When the TPC Trigger is issued, the online computer starts data taking.

### 3.4.3 Software Trigger

The TPC Trigger, previously mentioned, makes a loose \(z\)-constraint for charged tracks. A Software Trigger using a tighter constraint on the \(z\)-position was developed for high luminosity experiments, to reduce the rate of the Charged Trigger. This trigger requires two or more tracks coming from the beam interaction point by examining the timing signals from Time Projection Chamber. Though the Time
Figure 3.30: TPC wire configuration and the logic to remove noise.

Figure 3.31: TPC wire signal generated by a track from the event vertex.
Figure 3.32: The vertex decision in TPC Trigger.

Projection Chamber can measure spatial positions in three dimensions, it takes as long as 15 msec to digitize the timing signal to get the z-position. This long time makes a large dead time at every trigger decision. Therefore it is essential to reduce the trigger rate as much as possible to reduce dead time. A new Software Trigger was installed in the TOPAZ trigger system for the high luminosity experiments in February, 1990.

To make this trigger quick, four micro-processors are used in parallel. Fig. 3.33 shows the z distribution of charged tracks in triggered events. Here the solid histogram is the z distribution of triggered events with the Software Trigger, the dashed one is without this trigger. Thanks to this trigger, the vertex constraint was tightened and the rate of the Charged Trigger was reduced to 30 ~ 40% depending on the beam condition. The inefficiency of this trigger for two track events was obtained to be 0.5% by studying cosmic ray events. About 400 μsec is used for making this trigger. The dead time due to this trigger system is negligibly small (less than 0.3%)

3.5 Data aquisition system
Figure 3.33: The $z$ distribution of the tracks. The solid histogram is the $z$ distribution of software triggered events and the dashed one is that of TPC triggered events.
Figure 3.34: The TOPAZ data acquisition system.
The TOPAZ data acquisition system is shown in Fig. 3.34. The online computer, a VAX-11/780, takes the information from detectors, using a FASTBUS system. The information from detectors comes in the form of the electric analogue signals which are converted to digital signals by Analogue-to-Digital Converters (ADC) or Time-to-Digital Converters (TDC) very quickly on the FASTBUS system. To check the environmental condition of detectors, such as gas pressures, high voltages, temperatures and currents of each counter, a µPDP11 takes this information and sends it to the VAX-11/780 every few minutes. We also check the experimental conditions by displaying the sampling events on the screen of a high speed graphic terminal.

The DACU job sends these data to the TRISTAN central computer, a FACOM M-780, via an optical fiber cable at about 350 Kb/sec speed. These data are then reconstructed and analyzed. We will describe data processing in Chapter 5.
Chapter 4

Performance of the Luminosity Monitor

The luminosity measurement is very important for $e^+e^-$ colliding experiments. The luminosity is defined as the ratio of the event rate ($R$) to the cross section ($\sigma$) for a specific interaction as follows:

$$L = \frac{R}{\sigma (cm^{-2}s^{-1})}.$$  \hspace{1cm} (4.1)

In general, small angle Bhabha scattering ($e^+e^- \rightarrow e^+e^-$) is used for the luminosity measurement, because Bhabha scattering is a well known process. The QED differential cross section of lowest order is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16E^2} \frac{(3 + \cos^2 \theta)^2}{(3 - \cos \theta)^2},$$  \hspace{1cm} (4.2)

$$= \sim \theta^{-4} \text{ for small } \theta.$$  \hspace{1cm} (4.3)

Here $E$ is the beam energy, $\alpha$ is the fine structure constant and $\theta$ is the scattering angle. The differential cross section has sharp peak at small angle scattering. The Luminosity monitor was located at a small polar angle in order to catch a large number of the Bhabha events.

The main characteristics of the Luminosity Monitor are described briefly in Sec.3.2.7. In this Chapter, at first, we will describe the trigger system of the Luminosity Monitor. Then, the performance of the counters will be described. Finally, we will describe the Luminosity measurement.
4.1 Trigger system

The Luminosity Monitor consists of four sets of counters located symmetrically around the beam interaction point as described in Section 3.2.7 and as shown in Fig.3.14. The combination of a Defining \((D)\) counter and its opposite Complementary \((C)\) counter (for example \(D_1C_4\) etc.) defines the geometric acceptance of the Luminosity Monitor for Bhabha scattering. The corresponding Shower \((S)\) counter measures the energy of the scattered particle.

In order to detect back-to-back Bhabha events, we used two kinds of trigger. One is the Bhabha trigger \((T_{Bhabha})\). Bhabha events were taken by the Bhabha trigger. The other is the Background trigger \((T_{Background})\). The background is estimated using events which are taken by the Background trigger. We defined these triggers as follows:

\[
T_{Bhabha} = \sum_{i=1}^{4} T_i
\]

\[
with \quad T_i = (D_is_i) \cdot (C_{3-i}S_{3-i})
\]

\[
T_{Background} = \sum_{i=1}^{4} iT_i
\]

\[
with \quad iT_i = d(D_is_i) \cdot (C_{3-i}S_{3-i})
\]

Here \(i\) is the counter number as Fig.3.13 shown. \((D_is_i)\) represent the coincidence of the output signals from \(D_i\) and \(S_i\) counter and “\(\cdot\)” means the logical “AND”. \(d(D_is_i)\) represent the delayed \((D_is_i)\) signal of one cycle \((5\mu sec)\) per beam crossing. The trigger logic for \(T_i\) is shown in Fig.4.1 (a). The timing chart for \(T_{Bhabha}\) and \(T_{Background}\) are shown in Fig.4.1 (b) and (c), respectively.

4.2 Performance of counters

4.2.1 Pulse height distributions

The pulse height distributions of the \(D_1\) and \(C_1\) counter triggered by \(T_1\) are shown in Fig.4.2 (a) and (b), respectively. The thicknesses of plastic scintillators of \(D_1\) and \(C_1\) Counter are 5. and 8. mm, respectively. We can see the Landau distribution around 350 channels corresponding to the minimum ionization of high energy charged particles.

The scatter plots of the deposit energy in \(S_1\) and \(S_4\) counters for events which are taken by \(T_1\) trigger and \(dT_1\) trigger are shown in Fig.4.3 (a) and (b), respectively.
Figure 4.1: The trigger logic and the timing chart. (a) shows the trigger logic for LUM trigger and (b) shows the timing chart for Bhabha trigger and (c) for Background trigger.
Figure 4.2: Pulse height distribution for Defining and Complementary counters. (a) shows that for $D_1$ and (b) for $C_4$. 
Figure 4.3: The scatter plot of the deposit energy in the $S_1$ and $S_4$ counters for Bhabha triggered events (a) and for Background triggered events (b).
We can see the Bhabha events around 26 GeV in Fig.4.3 (a) and the backgrounds from the accidental coincidence events in Fig.4.3 (b).

The energy distribution of the $S_4$ counter is shown in Fig.3.15. It is given by projecting Fig.4.3 (a) and (b) when the deposit energy of $S_1$ is greater than 20 GeV. The histogram and solid circles in Fig.3.15 are the Bhabha and Background triggered events, respectively. The background triggered event distribution well reproduced the background distribution of Bhabha triggered events. The energy resolution for the shower counter was estimated using the Gaussian fit:

$$\frac{\sigma}{E} = 3.5\% \text{ at } \sqrt{S} = 52\text{GeV} . \quad (4.8)$$

### 4.2.2 Counter stability

It was very important to see the effects on the radiation damage for counters because the Luminosity Monitor was located near the beam-pipe. Fig.4.4 shows average pulse heights of the $D_1, C_1$ and $S_1$ counters for Bhabha events during a long period of time. The selection criteria for the Bhabha events will be described in the next section. The gain decrease was less than 10% and was no problem.

### 4.3 Luminosity measurement

#### 4.3.1 Bhabha event selection

From Fig.4.2 and ??fig:LUMbha, we selected Bhabha events according to the following condition:

1) The pulse heights of $C$ and $D$ counters are greater than 160 ADC channels.

2) The deposit energy of the $S$ counter is greater than $0.77E_{beam}$.

#### 4.3.2 Corrections for the luminosity measurement

The integrated luminosity for the TOPAZ detector, $L_{TOPAZ}$, was determined with the following formula:

$$L_{TOPAZ} = \frac{1}{\sigma_0(1 + \delta_{rad})} N_{Bhabha} \cdot \eta_{live} \cdot \eta_{acc} . \quad (4.9)$$

$\sigma_0$ is the cross section of the lowest order Bhabha scattering for the Luminosity Monitor region. At $\sqrt{s} = 52$ GeV, $\sigma_0$ is 46.7 nbarn.
Figure 4.4: Average pulse height of $D_1$, $C_1$ and $S_1$ counters for Bhabha events during long time from Jul. 27 in 1987 to Mar. 14 in 1988.
\( \delta_{\text{rad}} \) is the radiative correction factor up to the order \( \alpha^3 \), where \( \alpha \) is the fine structure constant for QED. Typical value of \( \delta_{\text{rad}} \) was \( \sim -0.07 \) at \( \sqrt{s} = 52 \) GeV.

\( N_{\text{Bhabha}} \) is the total number of Bhabha events, and is defined by

\[
N_{\text{Bhabha}} = \frac{(N_1 + N_4)}{2} + \frac{(N_2 + N_3)}{2},
\]

(4.10)

with \( N_i = N(T_i) - N(dT_i) \).

(4.11)

According to this definition of \( N_{\text{Bhabha}} \), we can reduce the effect of interaction point uncertainty.

\( \eta_{\text{live}} \) is the correction factor for the live time of TOPAZ data acquisition system (\( T_{\text{TOPAZ}} \)) and that of Luminosity Monitor (\( T_{\text{LUM}} \)) as follows:

\[
\eta_{\text{live}} = \frac{T_{\text{TOPAZ}}}{T_{\text{LUM}}}.
\]

(4.12)

Typical values have the range 0.95 \( \sim \) 1.05.

\( \eta_{\text{acc}} \) is the correction factor for accidental hits. The typical trigger rate is 0.5\( \sim \) 1.0 Hz, and the typical single counting rate of the \( D \) counter is \( \sim 10 \) kHz. When beam background increases, this single counting rate also increases, because soft \( \gamma \)-rays hit the \( D \) counter. Under the worst beam conditions, the single counting rate increases up to 200 KHz, which is the frequency of the beam crossing. The hits of soft \( \gamma \)-rays and the smaller angle Bhabha events out of the acceptance angle of Luminosity Monitor are accidental coincidence. By this accidental hits, the spurious Bhabha trigger was opened. We estimated this accidental ratio using a Monte Carlo simulation with a function of the single counting rate of the \( D \) and \( C \) counter as follows:

\[
\eta_{\text{acc}} = \frac{1}{\eta_{\text{single}}},
\]

(4.13)

\[
\eta_{\text{single}} = 1 + (0.39 \eta_D \eta_C + 1.12 \eta_D + 0.00026 \eta_C).
\]

(4.14)

\[
\eta_{D,C} = \frac{\text{Single counting rate of } D,C \text{ counter (KHz)}}{\text{Beam crossing frequency (200KHz)}}.
\]

(4.15)

Fig.4.5 shows the simulated \( \eta_{\text{single}} \). In the actual experiment the number of single counts of \( D \) and \( C \) counters were monitored in the CAMAC scales and read by the Micro PDP-11 and stored in the LUM data when the Bhabha trigger occurred. A typical value of \( \eta_{\text{acc}} \) is \( \sim 0.95 \).

### 4.3.3 Systematic error of luminosity measurement

The main components of the systematic error for luminosity measurement are as follows.
Figure 4.5: The simulated $\eta_{\text{single}}$. 
1) Effect of the energy threshold.

2) The position precision.

We use the $\sqrt{s} = 52$ GeV data for this error analysis.

1) **Effect of the energy threshold**

The threshold value of selecting Bhabha events for the $S$ counter has a chance to cause errors in luminosity measurement. We studied the deposit energy shape of the $S$ counter for experimental data and Monte carlo simulation data. In this simulation the Bhabha events were generated with radiative correction up to $O(\alpha^3)$, and simulated the shower process by the Electromagnetic Shower program EGS4. We also took into account the dependence of pulse height on the vertical incident particle position, $\sim 1.5\%$, described in Ref.[30]. For the correction factor $\eta_{acc}$, $\eta_D = \eta_C = 0.08$ is used. This value was obtained as the averaged value of the experimental results. The simulation result is shown by the solid circle in Fig.4.6 with the experimental data. We studied the threshold effect at $\pm 2$ GeV of our threshold value, $0.77E_{beam}(=20$ GeV), as a ratio $N_{Bhabha}/N_{M.C.}$, which is shown in Fig.4.7 This caused an uncertainty of in luminosity of at most

$$\frac{\Delta L_1}{L} = \pm 2.3\%.$$  

(4.16)

2) **The position precision**

The distance between two $D$ counters facing each other in the radial direction was measured to be $d \pm \Delta d = 78$ mm $\pm 200$ $\mu$m before installation. The systematic error of the luminosity due to the precision of the position measurement is given as follows:

$$\frac{\Delta L_2}{L} \simeq 4 \frac{\Delta d}{d} = \pm 1.0\%.$$  

(4.17)

The relative error of the luminosity due to the difference ($\delta$) between the radial center of the $D$ counters and actual beam line position is expressed as [41]

$$\frac{\Delta L}{L} = 10(\frac{\delta}{d})^2.$$  

(4.18)

This value can be estimated using the ratio $R$, which is defined as

$$R \equiv \frac{N_1 + N_3}{N_2 + N_4}.$$  

(4.19)
Figure 4.6: The Monte Carlo simulation for Bhabha events.
Figure 4.7: The energy threshold effects for luminosity measurement.

And this relative error is expressed by

\[ \frac{\Delta R}{R} \simeq 8 \frac{\delta}{d} . \]  (4.20)

Fig. 4.8 shows the ratio \( R \). We estimated at most

\[ \frac{\Delta R}{R} = \pm 20\% \]  (4.21)
\[ \frac{\Delta L_3}{L} = \pm 0.6\% . \]  (4.22)

We studied the effect of the horizontal displacement by Monte Carlo simulation, and found it to be about half that of the radial one. Therefore we estimated the horizontal displacement effect to be the same as that of radial one,

\[ \frac{\Delta L}{L} = \pm 0.6\% . \]  (4.23)

The total systematic error was estimated to be

\[ \frac{\Delta L}{L} = \pm 2.6\% \text{ at } \sqrt{s} = 52\text{GeV} . \]  (4.24)
Figure 4.8: The measured the ratio $R(\equiv (N_1 + N_2)/(N_3 + N_4)$.
4.3.4 Results of luminosity measurement

According to the collider theory, Luminosity($L$) is expressed by

$$L \sim I_+ I_-,$$  \hspace{1cm} (4.25)

where $I_+$ and $I_-$ are the positron and electron beam currents, respectively. Fig.4.9 shows the measured luminosity for every three minutes as a function of $I_+ I_-$ for one beam fill with the line fitted to data by the least square.

The integrated luminosity which was measured by the LUM is shown in Table 4.1 with the luminosity measured by ECL. They agree within error.
<table>
<thead>
<tr>
<th>$\sqrt{S}$ (GeV)</th>
<th>LUM $(pb^{-1})$</th>
<th>ECL $(pb^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>3.63 ± 0.10</td>
<td>3.60 ± 0.04</td>
</tr>
<tr>
<td>55</td>
<td>2.95 ± 0.08</td>
<td>2.88 ± 0.04</td>
</tr>
<tr>
<td>56</td>
<td>5.64 ± 0.15</td>
<td>5.64 ± 0.05</td>
</tr>
</tbody>
</table>

Table 4.1: Integrated Luminosity
Chapter 5

Raw Data Processing

In this chapter we will describe how to get the physical information, such as momentum, energy and positions of the detected particles, from the electronic digital information for each counter.

5.1 Flow of Data Processing

The flow of the TOPAZ data processing is shown in Fig. 5.1 The data on the VAX are transferred to the temporary disk buffer of the FACOM M-780 by the DACU. These data are copied onto magnetic cassette tapes and converted to the TOPAZ standard data format to match our data handling system called the TRISTAN Bank System (TBS). The formatted data are stored as raw data on magnetic cassette tapes. The capacity of this tape is ~ 280 Mbytes. For typical beam conditions, one format tape stores about 5000 ~ 8000 events. We can access a large number of tapes easily and quickly thanks to an automatic mounting system.

The contents of raw data are digital signals from Analog-to-Digital Converters (ADC) or Time-to-Digital Converters (TDC). These signals are used to reconstruct the physical information in two steps. One step is called “Reduction” and the other is “Production”.

In the Reduction step, the size of the data is reduced by rejecting obviously non-interesting events such as beam-gas events. The raw data of the digital signals are converted as follows. For calorimeters, ADC values are calibrated to energy, and for tracking chambers, TDC’s values are calibrated to spatial information for each wire. In the Production step, the converted data by the Reduction step are used to reconstruct following physical observables: for calorimeters the energy and position of each energy cluster and for tracking chamber drift time and hit point.
Figure 5.1: The flow of the TOPAZ data processing.
The production data are stored in Data Summary Tapes (DST). The DST data are further calibrated for beam condition, for example, the time dependence of the gain for the Forward Calorimeter. They are stored in the tapes called DSTPASS2.

5.2 Event Reduction

To reduce the raw data size non-interesting data are removed with following conditions:

1. We check the trigger timing measured from beam crossing for the energy triggered events,
   1) For Barrel Calorimeter, $360 \text{ nsec} < t < 390 \text{ nsec}$,
   2) For Endcap Calorimeter, $300 \text{ nsec} < t < 700 \text{ nsec}$,

2. For TPC triggered events, the number of tracks with $|z| \leq 10 \text{ cm}$ in the Time Projection Chamber,

where $|z|$ is the absolute value of the closest approach to the z axis for each track. These conditions are loose enough to retain all interesting events.

5.3 Event Reconstruction

5.3.1 Cluster Reconstruction

Since the electromagnetic showers in the calorimeter, which were made by electrons, positrons and photons, expand in several counters, we can measure the energy deposits in these counters. In order to know the energy and position for incident particles, we have to reconstruct energy clusters from the energy deposit. We used energy cluster information for tagging the electron or positron in this analysis. In the following sections, we describe the reconstruction method for the Barrel Calorimeter, Endcap Calorimeter and Forward Calorimeter.

Cluster Reconstruction for the Barrel Calorimeter

The Barrel Calorimeter consists of 4300 lead glass counters (Sec. 3.2.5). The raw data from the Barrel Calorimeter are digitized pulse heights from lead glass counters. In the Reduction step, each digitized value is converted into energy deposit for each
counter. These converted data are reconstructed to the energy and position data using following methods:

1) Find a counter \( (C_1) \) having the largest energy deposit \( (E_1) \),

2) Pick up the neighbouring four counters \( (C_2) \) of \( C_1 \).

If the energy deposit \( (E_2) \) in \( C_2 \) is greater than 0.65\( E_1 \), this counter \( (C_2) \) is grouped in the same cluster as \( C_1 \).

3) Pick up the neighbouring three counters of \( C_2 \).

If the energy deposit \( (E_3) \) in \( C_3 \) is greater than 0.08\( E_1 \) and 1.20\( E_2 \), \( C_3 \) is grouped in the same cluster as \( C_1 \).

This procedure is repeated until all counters whose energy deposits are greater than 45 MeV are used.

The positions for clusters are calculated as the averaged counter positions weighted by their energies in the \( r - \phi \) plane:

\[
X = \frac{\sum_i X_i \cdot E_i^a}{\sum_i E_i^a}
\]  

(5.1)

where \( X_i \) is the \( r \) or \( \phi \) of \( i \)-th lead glass counter, \( E_i \) is the energy of the \( i \)-th counter, and \( a \) is an empirical constant being set to 0.26 for \( z \) and 0.34 for the \( \phi \) direction. The \( r \) of the cluster is calculated from the depth of the shower maximum as if a photon coming from the interaction point.

**Cluster Reconstruction for the Endcap Calorimeter**

The Endcap Calorimeter consists of wire proportional tubes inserted between Pb absorbers, and has a three-stage tower structure of cathode pads (Sec. 3.2.9). The raw data of the Endcap Calorimeter consist of digitized pulse heights counts of the towers. In the reduction step, these digitized data are converted into deposit energies tower-by-tower. The clustering method for the Endcap Calorimeter is simpler than that of the Barrel Calorimeter.

1) Find the tower with the maximum energy deposit.

2) The towers within an angle of 0.16 radian around the highest tower is grouped into the same cluster of the highest one.

This procedure is repeated until all towers whose energy deposit are greater than 50 MeV are used. The cluster position is determined by the same method as the Barrel Calorimeter using Eq. (5.1). Here \( X_i \) is the coordinate of \( i \)-th tower in the \( \theta - \phi \) plane, and the parameter \( p \) is set to be 1 for both the \( \theta \) and \( \phi \) directions.
Cluster Reconstruction for the Forward Calorimeter

The clustering procedure for the Forward Calorimeter is different from that for the Barrel and Endcap Calorimeters. The Forward Calorimeter consists of BGO crystal counters for energy measurements and Si strip detectors for position measurements (Sec.3.3.3). The raw data of the Forward Calorimeter are the digitized pulse heights from BGO and Si-strip detectors. They are converted into energy deposits in the Reduction step.

**BGO Clustering** The consecutive counters whose energy deposits are greater than $E_{\text{threshold}}$ are grouped in the same clusters. The threshold energy ($E_{\text{threshold}}$) is 150 MeV, which is three times higher than the noise level.

**Si-strip Clustering** The clustering method for Si-strip is the same as that for the BGO crystal. The consecutive strips on the same Si-pads whose energy losses are greater than 30 KeV are grouped into the same clusters. Fig. 3.26 shows the energy loss distribution for minimum ionizing particles (29 GeV electrons in Bhabha events).

The position of the Si-strip cluster is determined as follows.

1) If there is only one strip hit in a cluster, the position of the strip is given as the cluster position.

2) If there are a few hits in a cluster,
   
   i) If the number of hits is odd, the position of the central strip in the cluster is given as the cluster position.
   
   ii) If the number of hits is even, the central two strips in the cluster is used at random to determine the position. The position of the strip is the center value of $(r, \phi, z)$, and their error is determined by the width of the strip and the counter.

**The position of BGO Cluster** The determination method of the BGO cluster position is as follows:

1) Do the BGO clustering,

2) Find the BGO counter which has the maximum energy deposit,

3) Check the Si-strip cluster in front of the BGO counter having maximum energy deposit,
Figure 5.2: A block diagram of the Time Projection Chamber analysis.

i) If there is no Si-strip cluster, the center position of the BGO is given as the cluster position.

ii) If there is only one Si-strip cluster, its position is given as the cluster position.

iii) If there are few Si-strip clusters, the position of the cluster is determined in the same way as the Si-strip clustering.

5.3.2 Track Reconstruction for Time Projection Chamber

Track reconstruction [42] for the TPC is done in three steps, i.e., Space Point Making, Track Finding and Track Fitting. In the Space Point Making step, all signals from the Time Projection Chamber are converted to a set of three dimensional hit points. Then, these hit points are associated to tracks. This step is called “Track Finding”. Then these tracks are converted to track parameter vectors. This step is called “Track Fitting”.

The flow chart of the analysis for the Time Projection Chamber is shown in Fig. 5.2. In following, we will describe the method of track reconstruction in detail.
**Space Point Making** Before describing the Space Point Making, we define the local coordinate system $(\xi, \eta, z)$ attached to each sector of the Time Projection Chamber. As Fig. 5.3 shows, the $\xi$ axis is along the sense wire direction, the $\eta$ axis points in the radial direction at the center line of the sector, and the $z$ axis is along the beam line.

The pad data consist of pad numbers representing the pad locations, bucket numbers corresponding to the $z$ coordinates, and pulse heights associated with the CCD (Charge Coupled Device) buckets. To find hit points we make a histogram in $(\xi - z)$ plane using pad data. The binnings of this $(\xi - z)$ histogram are determined by the pad width and the bucket size, respectively. We search for peaks in this histogram. If we find a peak, we calculate the $\xi$ and $z$ coordinates by fitting the pulse heights of contributing buckets to parabolas in both $\xi$ and $z$. Although we already knew the $\eta$ position from the pad location, the position is corrected for Landau fluctuation of sense wire signals, because five sense wires near the pad row contribute to the pad pulse height. Using the pulse heights of relevant wires, we calculate the $\eta$ position as an appropriately weighted average of the $\eta$ position for these wires.

**Track Finding** In order to find tracks as quickly as possible, the track finding is done by two steps. First the high momentum tracks are found from reconstructed space point data (“Level-1 Track Finding”), because it is easy to find these tracks.
Then the space point data of these tracks are removed. Next we find the low momentum or secondary tracks, which are more difficult to find, from remaining space point ( "Level-2 Track Finding" ).

**Level-1 Track Finding** We use a two-dimensional histogram in the \( \phi - Z(R_{ref}) \) plane to find a track from reconstructed space point data quickly. Here \( \phi \) is the azimuthal angle and \( Z(R_{ref}) \) is defined by:

\[
Z(R_{ref}) = Z \times \left( \frac{R_{ref}}{R} \right)
\]

with \( R_{ref} \) is fixed at some appropriate value. If several hit points corresponds to the same track, a cluster is made by these hit points in this histogram. For example, high momentum tracks from the interaction point falls in the same bin. Next we search for bins containing hits points over a given threshold. When such a bin is found, we chose three space points which span the largest lever arm and check if they are on a single helix. Then these rough helix parameters are extrapolated in order to find other bins containing associated hits. The associated hits are removed from this histogram. This process is repeated using the lower threshold.

**"Level-2 Track Finding"** We chose the bins containing the largest number of remaining points. Then we take the innermost hit point as a pivot. Using this pivot, a new \( \phi - Z(R_{ref}) \) histogram is created with a larger bin width so as to contain more than one hit in a single bin for the low momentum tracks. We find a bin above threshold in the same way as for Level - 1.

**Track Fitting** The found tracks from space points are fitted to the following helix parameters:

\[
\begin{align*}
X &= X_0 + d\rho \cos \phi_0 + \frac{1}{\kappa} (\cos \phi_0 - \cos (\phi + \phi_0)) \\
Y &= Y_0 + d\rho \sin \phi_0 + \frac{1}{\kappa} (\sin \phi_0 - \sin (\phi + \phi_0)) \\
Z &= Z_0 + dZ + \frac{1}{\kappa} \tan \lambda.
\end{align*}
\]

The parameter vector to fit is:

\[
\alpha = \begin{pmatrix}
    d\rho \\
    \phi_0 \\
    \kappa \\
    dZ \\
    \beta
\end{pmatrix} = \begin{pmatrix}
    d\rho \\
    \phi_0 \\
    \frac{1}{\kappa} \\
    dZ \\
    \tan \lambda
\end{pmatrix},
\]
where \((X_0, Y_0, Z_0)\) is usually taken to be the innermost hit point, \(\rho\) is the helix radius, \(\lambda\) is the dip angle measured from the \(XY\) plane, \(d\rho\) and \(dZ\) are corrections to \((X_0, Y_0, Z_0)\), respectively, and \(\phi\) is the deflection angle measured from \((X_0, Y_0, Z_0)\). These parameters are defined in Fig. 5.4. The charge of the track is determined by the sign of \(\kappa\). To find \(\alpha\) we define the following \(\chi^2\):

\[
\chi^2 = \sum_i \left( \frac{\xi_i - \xi(\eta_i, \alpha)}{\sigma_{\xi_i}} \right)^2 + \left( \frac{Z_i - Z(\eta_i, \alpha)}{\sigma_{Z_i}} \right)^2 .
\]

The performance of the track reconstruction was studied by using TOPAZ simulator data. The momentum resolution obtained as \(\sigma_{p_i}/P_T = 1.0\%\). The tracking efficiency with a transverse momentum cut at 0.2 Gev is 95.3\% for primary tracks and 94.2\% including secondary tracks in multihadron events. The efficiency plateau does not reach 100\% because of sector boundary and shallow track effects.

**Track Refitting** The space points of each track are corrected for distortions and re-fitted after all tracks are found. Although the gating grid of the Time Projection Chamber suppresses the electric field distortion as previously mentioned, a small amount of distortion still remains. Because of this effect, the reconstructed tracks are shifted systematically. To remove the effect, the electrostatic distortion is parametrized by an empirical formula. These parameters are determined from the cosmic ray data. After the Trigger Chamber was installed, the distortion was corrected completely with the help of the Trigger Chamber. The corrected space points are refitted to a new helix.
Figure 5.4: The parameterization for a charged track of Time Projection Chamber
Chapter 6

Event Selection

In this chapter, we will describe the selection criteria of single-tag events.

6.1 Event selection of single-tag events

In events which were selected, one electron (or positron) in the final state of the reaction $e^+e^- \to e^+e^- + \text{hadrons}$ was detected by either one of calorimeters; i.e., the Forward Calorimeter (FCL), Endcap Calorimeter (ECL) and Barrel Calorimeter (BCL) in the TOPAZ detector. We call these events “FCL events”, “ECL events” and “BCL events”, respectively. The momentum transfer, $Q^2$, of the virtual photon was determined from the energy and the scattering angle of the tagged electron measured by these devices.

The momenta of charged particles in this reaction are measured by TPC, and the energy of the neutral particles are measured by ECL and BCL.

In the single-tag condition, one scattered electron is missing along the beam axes carrying approximately the beam energy. Single-tag events therefore have the following characteristics. 1) The observed energy of the hadron system is less than twice the beam energy ($2E_{\text{beam}}$), and 2) the total longitudinal momentum does not balance, but 3) the vector sum of the transverse momentum of all particles should be balanced.

Making use of these characteristics, the selection criteria were determined so that single-tag events could be selected with good efficiency and the background from other reactions would become as small as possible.
<table>
<thead>
<tr>
<th>Tagging device</th>
<th>FCL</th>
<th>ECL</th>
<th>BCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^\pm$ tag</td>
<td>$E_{tag}/E_{beam}$</td>
<td>$\geq 0.32$</td>
<td>$\geq 0.32$</td>
</tr>
<tr>
<td></td>
<td>$</td>
<td>\cos \theta_{tag}</td>
<td>$</td>
</tr>
<tr>
<td>$e^\mp$ anti-tag</td>
<td>$E_{\text{cluster}}/E_{beam}$</td>
<td>$\leq 0.2$</td>
<td>$\leq 0.2$</td>
</tr>
</tbody>
</table>

### Hadron selection

- $N_{\text{charged}}$
  - (exclude tagged $e^\pm$)
  - $|\cos \theta| < 0.85$
  - $|R| \leq 5\text{cm}$
  - $|z| \leq 15\text{cm}$
  - $|P_t| \geq 0.15\text{GeV}/c$

- $|V_2|$ (cm): $\leq 5.0$

- $W_{\text{vis}}$ (GeV):
  - $0.87 < |\cos \theta_{ECL}| < 0.98$
  - $E_{ECL} > 700\text{MeV}$
  - $|\cos \theta_{BCL}| < 0.8$
  - $E_{BCL} > 500\text{MeV}$

### Background rejection

- $\sum P_t/E_{beam}$: $\leq 0.2$
- $(E_{vis} - E_{tag})/E_{beam}$: $\leq 0.65$
- $(\sum P_z)/E_{beam}$: $\geq 0.7$
- $\sum |P^{\leq \theta_0}|$ (GeV/c): $\leq 0.7$
- $\theta_0$ (rad.): $= 0.4$

### Table 6.1: Selection criteria of single-tag $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ reaction.

#### 6.1.1 Selection criteria for single-tag events

The events tagged by each device were selected separately but with the almost similar selection criteria. The selection criteria for each device are summarized in Table 6.1. For example, the single-tag events tagged by ECL ("ECL events") were selected by the following criteria.

- **$e^\pm$-tag**

1. The energy of the tagged electron (or positron) must be greater than $0.32 \ E_{beam}$ in the angular range $0.89 \leq |\cos \theta_{tag}| \leq 0.96$.

2. No additional clusters with energy exceeding $0.2 \ E_{beam}$ must be observed in anywhere. This condition excludes double-tag events.
• Hadron selection criteria

3. At least 3 charged tracks coming from the interaction point must be observed in the TPC. To select the tracks from the $e^+e^-$ interaction point, we imposed the following criteria on each tracks.

   i) The closest distance to the beam axis must be less than 5 cm in the radial direction and less than 15 cm in $z$ (beam) direction.

   ii) The transverse momentum of the track must be greater than 0.15 GeV/c.

4. The event vertex position along the beam line $|V_z|$ must be less than 5 cm.

5. The visible mass of the hadron system ($W_{vis}$) must be greater than 0.2 GeV/c$^2$. Where $W_{vis}$ is calculated using all charged and neutral particles. The pion mass is assumed for all charged particles. Neutral clusters, whose energy was greater than 700 MeV and 500 MeV for the ECL and the BCL, respectively, were used in this $W_{vis}$ calculation in order to avoid multiple counting of the contribution from charged particles in the calorimeters and to reject noise in the calorimeters.

• Criteria to reject background

6. The vector sum of the transverse momenta ($\sum \mathbf{P}_t$) must be less than 0.3 $E_{beam}$.

7. The total visible energy minus the energy of the tagged electron; $\{E_{vis} - E_{tag}\}$, should be less than 0.7$E_{beam}$. Where $E_{vis}$ includes the contribution from charged particles measured by TPC and neutral particles measured by the BCL and ECL.

8. The longitudinal momentum imbalance projected along the direction of the tagged electron, $\{ (\cos \theta_{tag}/|\cos \theta_{tag}|) \sum \mathbf{P}_z \}$, must exceed 0.6 $E_{beam}$.

9. The tagged electron must be isolated. This criterion is imposed by the requirement that the sum of the absolute value of the momenta within a 0.4 radian cone with respect to the direction of the tagged electron (or positron), is less than 700 MeV.
<table>
<thead>
<tr>
<th>Tagging device</th>
<th>FCL</th>
<th>ECL</th>
<th>BCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; Q^2 &gt;$ (GeV$^2$)</td>
<td>9.7</td>
<td>87.</td>
<td>338.</td>
</tr>
<tr>
<td>$&lt; \sqrt{s} &gt;$ (GeV)</td>
<td>58.0</td>
<td>57.9</td>
<td>57.9</td>
</tr>
<tr>
<td>Luminosity(pb$^{-1}$)</td>
<td>89.3 ± 3.6</td>
<td>113.9 ± 4.6</td>
<td>113.9 ± 4.6</td>
</tr>
<tr>
<td>Observed events(events)</td>
<td>1997 ± 45</td>
<td>246 ± 16</td>
<td>32 ± 6</td>
</tr>
</tbody>
</table>

Table 6.2: The results of single-tag event selection for experimental data.

In order to show the effects of each criterion, various distributions of ECL events are shown in Fig. 6.1, where the data in the figures have undergone all cuts except the one in question. Fig. 6.1 (a) shows the vertex $z$ distribution. Fig. 6.1 (b), (c) and (d) show the distributions of visible mass of the hadron system, the vector sum of the transverse momenta of all charged and neutral particles and the visible energy of the hadron system, respectively. The distributions of the total longitudinal momenta are shown in Fig. 6.1 (e) and the sum of the absolute values of the momenta around the tagged electron are shown in Fig. 6.1 (f). The peak around 0.7 in Fig. 6.1 (e) shows the single-tag events and that around 0.25 is from background events from one photon annihilation processes. Quantitative study of the backgrounds will be discussed in Section 7.2. The selection efficiency depends on this cut (cut 8).

The effects of each selection criterion for the FCL events are shown in Fig. 6.2 (a) ~ (e).

The number of events which survived these criteria are 1997, 246 and 32 for FCL events, ECL events and BCL events, respectively. ECL events and BCL events were selected from the experimental data corresponding to 113.9 pb$^{-1}$ integrated luminosity at the center of mass energy ($\sqrt{s}$) from 52 GeV to 61.4 GeV. FCL events were selected from 89.3 pb$^{-1}$ experimental data at $\sqrt{s} = 58$ GeV. The results are summarized in Table 7.2.

### 6.2 Backgrounds

In this section, we discuss possible background processes which may contribute to the single-tag two-photon event sample. A more quantitative estimation of these background contributions will be discussed in Section 7.2.
Figure 6.1: The effects of each selection criterion for ECL events. The data in the figures have undergone all cuts except the one in question. The figures show (a) : vertex $z$ position, (b) : visible mass of the hadron system, (c) : the sum of the transverse momentum vectors, (d) : the visible energy of the hadron system, (e) : the total longitudinal momenta and (f) : the sum of the absolute value of the momenta around the tagged electron. The cut position is shown by arrow in each figure.
Figure 6.2: The effect of each selection criterion for FCL events. The data in the figures have undergone all cuts except the one in question. The figures show (a) - the vertex $z$ position, (b) - visible mass of the hadron system, (c) - the sum of transverse momentum vectors, (d) - the visible energy of the hadron system and (e) - the longitudinal momentum distribution. The arrow shown in each figure shows the cut position to select the single-tag two-photon events.
The following processes will be presumed as backgrounds for the single-tag events.

1) $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$
2) Inelastic Compton scattering
3) $e^+e^- \rightarrow q\bar{q}, q\bar{q}\gamma$
4) $e^+e^- \rightarrow \tau^+\tau^-, \tau^+\tau^-\gamma$
5) Beam-gas interactions

1) $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$

This process is presumed that to be the main background for single-tag events. The Feynman diagrams of this process are depicted in Fig. 6.3. The process consists of two kinds of diagrams, i.e., the two photon process (a) and the bremsstrahlung diagrams (b). The process $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ show similar signatures to the hadron events in the two photon process since the $\tau$ also decays into final states containing hadrons. As shown in Section 7.2 in detail, the contamination of this process was estimated to be about 4.2% for FCL events and 10.4% for ECL events and 6.2% for BCL events.

2) Inelastic Compton scattering

Inelastic Compton scattering is depicted in Fig. 6.4. A photon emitted by one electron interacts with the other. The Feynman diagrams for this process is shown in Fig. 6.3 (b). The contamination of this processes was estimated from the calculation of the Feynman diagrams in Fig. 6.3 (b).

3) $e^+e^- \rightarrow q\bar{q}, q\bar{q}\gamma$

The hadronic events in one-photon annihilation process shown in Fig. 6.5 are another source of background.

These processes can be suppressed by selection criteria 7,8 and 9. Criterion 8 is found to be very effective in rejecting these one-photon annihilation events because the total longitudinal momentum is balanced in one-photon events. The main background from this process comes when a high energy radiative
Figure 6.3: The Feynman diagrams of $e^+e^- \rightarrow e^+e^- \tau^+\tau^-$ and $e^+e^- \rightarrow e^+e^- q \bar{q}$ process.
photon or a photon from $\pi^0$ is mistaken for a tagged electron in the no-tracking region.

4) $e^+e^- \rightarrow \tau^+\tau^-, \tau^+\tau^-\gamma$

This process (Fig. 6.5) is also a background source in principle. However, $\tau$ pair events are characterized by transverse momentum imbalance because $\tau$ always decays with a high momentum neutrino which cannot be detected. Selection criterion 6, the transverse momentum balance, can suppress the contamination of this process effectively.

5) **Beam-gas interactions**

If one beam interacts with the gas remaining in the beam pipe, the total energy of this interaction can be almost the beam energy. Beam-gas events show similar signatures to two photon events. But this contamination can be estimated from the event vertex distribution ($V_Z$) along the beam direction. Since the $V_Z$ should be flat for beam-gas events. Beam-gas interactions which occurred outside of the beam-beam interaction region are suppressed by the selection criterion 4. The contamination due to beam-gas events was estimated using the side band events in the $V_Z$ distribution as shown in Section 7.2.
Figure 6.5: Hadron production and tau pair production via one photon process.
Chapter 7

Experimental results and comparison with Monte Carlo predictions

In Chapter 6, we showed the selection criteria for single-tag two-photon events. In order to estimate effects of the detector, i.e. limited acceptance and the finite resolution of the detector, we need a Monte Carlo simulation. In this Chapter, we will describe, in detail, the Monte Carlo event generation and detector simulation. Then, the simulated data will be compared with the experimental data.

7.1 Monte Carlo Simulation

In order to compare the experimental data with theoretical predictions, it is very important to know our detector acceptance and resolution accurately, since the TOPAZ detector does not cover all solid angle and the resolution is finite. The Monte Carlo simulation is very useful to estimate them.

Fig.7.1 shows the flow chart of the data processing for Monte Carlo data and that of the experimental data. As mentioned in Chapter 5, in the reduction step, the size of the raw data is reduced and the raw data is converted to the energy information in the calorimeters or the space point information in the tracking devices. In the event reconstruction step, the momenta of charged particles and the energy of neutral particles are obtained using the information
Figure 7.1: The flow of Monte Carlo data analysis and the flow of experimental data analysis.
from various detectors. In the Monte Carlo simulation, events are first generated according to theoretical formulae. Then, the generated events are passed through detector simulation programs, where the response of the detector (i.e. resolutions and acceptance) are simulated as close to the real response of the detectors as possible. The output data from the simulator have the same format as those of the real data. Simulated events are therefore reconstructed and are analyzed by the same programs as the one used for experimental data.

In following subsections, details of the event generation, the detector simulation and the trigger simulation are described.

7.1.1 Event Generation

In this section, we will describe event generation for the process $e^+e^- \rightarrow e^+e^- + hadrons$ in the single-tag condition. The event generation for the process is carried out in the 3 steps as shown in Fig.7.2. In the first Step, the process $e^+e^- \rightarrow e^+e^- + hadrons$ is generated according to the general formula given in Eq. (2.19). In order to simulate the angular distribution of the hadron system, $W$, is decayed into a “$q\bar{q}$” pair in Step 2 and the quarks are fragmented into hadrons using the LUND (version 6.3) program [15] in the Step 3. Details of each step will be described in the following paragraphs.

Step 1: Event generation for the process $e^+e^- \rightarrow e^+e^- + hadrons$

The $e^+e^- \rightarrow e^+e^- + hadrons$ events are generated according to the theoretical formula given in Eq. (2.19), in the first step. In the program, the tagged electron energy $E_{tag}$ and the scattering angle $\theta_{tag}$ are used instead of the scaling variables of $x$ and $y$. Using these variables, the Eq. (2.19) are rewritten as follows:

$$
\frac{d\sigma}{dE_{tag}d\theta_{tag}dz} = \frac{4\pi\alpha^2 E_{tag}}{Q^4y} \left\{ \left(1 + (1 - y)^2\right) F_2^q(x, Q^2) \right\} f_{\gamma/e}(z) \quad (7.1)
$$

with

$$
\begin{align*}
    y &= 1 - \left( \frac{E_{tag}}{E_{beam}} \right) \cos^2(\theta_{tag}/2) \\
    z &= E_{\gamma}/E_{beam}
\end{align*} \quad (7.2)
$$

where $E_{\gamma}$ is the energy of the target photon and $f_{\gamma/e}(z)$ is the target photon flux factor described in Eq. (2.20). The explicit model of the photon structure function $F_2^q$ used here is described below.
Figure 7.2: Outline of single-tag event generation.

**Model of the photon structure function**

The photon structure function $F_2^\gamma$ is assumed to be decomposed into a hadron-like part and point-like part:

$$F_2^\gamma(x, Q^2) = F_2^{\gamma,HAD}(x, Q^2) + F_2^{\gamma,PL}(x, Q^2).$$  \hspace{1cm} (7.3)

The hadron-like contributions are well described by the Vector Meson Dominance (VMD) model, because neutral vector mesons have the same spin and parity as those of the photon. The most commonly used form

$$(F_2^{\gamma,HAD})_{VMD} = 0.2 \alpha(1 - x).$$ \hspace{1cm} (7.4)

is taken.

The point-like contributions are assumed to be given either by the QPM expression (Eq. (2.23)) or by the FKP expression (Eq. (2.38)). In the FKP expression, there are two free parameters. One is the QCD scale parameter $\Lambda$ and the other is the cut-off parameter $p_t^0$. We used $\Lambda = 0.2$ GeV and $p_t^0 = 0.5$ GeV/c. The mass of the quarks are assumed to be $m_u = m_d = 325$ MeV and $m_s = 500$ MeV. The contributions of heavy quarks are taken into account.
using the QPM expression given in Eq. (2.21). The masses of heavy quarks are assumed to be 1.6 GeV and 5 GeV for the $c$ quark and $b$ quark, respectively.

In order to generate the events according to Eq. (7.1), the differential cross section should be integrated numerically in the kinematically allowed region. For this purpose, we have used the program packages BASES and SPRING [43]. In the BASES step, the differential cross section is integrated numerically. In the SPRING step, $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ events are generated by the Monte Carlo method using the results from BASES.

**Step 2: The angular distribution of the hadron system**

In the QPM and QCD, the angular distribution of the produced quarks are expected to be similar to the one given in the reaction $\gamma \gamma \rightarrow \mu^+ \mu^-$, as follows:

\[
\frac{1}{\sigma_{\gamma \gamma \rightarrow \mu^+ \mu^-}(s)} \frac{d\sigma_{\gamma \gamma \rightarrow \mu^+ \mu^-}(s)}{d\Omega^*},
\]

where $\Omega^*$ is a solid angle in the center of mass system of the $\gamma \gamma$ system and $\sigma_{\gamma \gamma \rightarrow \mu^+ \mu^-}(s)$ is the total cross section for the reaction $\gamma \gamma \rightarrow \mu \mu$ at the center of mass energy $\sqrt{s}$ of the $\gamma \gamma$ system. This angular distribution is normalized to 1. Here the differential cross section and the total cross section of the $\gamma \gamma \rightarrow \mu \mu$ process are given [46] as follows:

\[
\frac{d\sigma_{\gamma \gamma \rightarrow \mu^+ \mu^-}}{d\Omega^*}(s) = \frac{\alpha^2}{2s} \left( 1 - \frac{4m^2}{s} \right)^{\frac{1}{2}} G_\mu(E_\mu, \theta^*)
\]

\[
G_\mu(E_\mu, \theta^*) = 2 + 4 \left( 1 - \frac{m^2}{E^2_\mu} \right) \frac{1 - m^2/E^2_\mu}{[1 - (1 - m^2/s)\cos^2 \theta^*]^2}
\]

\[
\sigma_{\gamma \gamma \rightarrow \mu^+ \mu^-}(s) = \frac{4\pi\alpha^2 s}{s} \left[ 2 + \frac{8m^2}{s} - \frac{16m^4}{s^2} \right] \ln \left\{ \frac{\sqrt{s}}{2m_\mu} + \left( \frac{s}{4m^2_\mu} - 1 \right)^{\frac{1}{2}} \right\}
\]

\[
- \left( 1 - \frac{4m^2}{s} \right)^{\frac{1}{2}} \left( 1 + \frac{4m^2}{s} \right)
\]

where $m_\mu$ and $E_\mu$ are the mass and energy of muon, respectively, and $\theta^*$ is the angle between one of the incident photons and an outgoing $\mu^\pm$ in the $\gamma \gamma$
center of mass system. These formulae are used to generate the perturbative part of $F_2^\gamma$.

For the VMD part, we use the exponential forms since the VMD is the soft process and the particles are expected to be produced at small scattering angles dominantly. We assumed the following angular distribution,

$$\frac{d\sigma}{dP_t^{*2}} = 3e^{-3P_t^{*2}} \quad (7.8)$$

$$P_t^{*} = P^* \sin \theta^* \quad (7.9)$$

where $P^*$ and $\theta^*$ are the momentum and scattering angle of produced quarks in the rest frame of the hadron system, respectively. The factor 3 normalizes this angular distribution to 1. The Eq. (7.8) can be rewritten as follows:

$$\frac{d\sigma}{d\cos \theta^*} = 6P^{*2}|\cos \theta^*|e^{-3P^{*2}}. \quad (7.10)$$

We used this angular distribution for the VMD part.

**Step 3 : Quark fragmentation**

The “quarks” generated in the previous are fragmented into hadrons by the LUND (version 6.3) program. In the LUND program, quarks are fragmented into hadrons according to the LUND symmetric function \[ f(z) \propto \frac{1}{z}(1-z)^a \exp\left(-\frac{bm^2_T}{z}\right), \quad (7.11) \]

where $z$ is the fraction of $E + p_T$ between the quark and the hadron, $m_T$ is the transverse mass of the hadron system, and $a$ and $b$ are the fragmentation parameters. The transverse momentum $p_T$ of the hadron is described by the Gaussian function

$$f(p_T) \propto \exp\left(-\frac{p_T^2}{2\sigma_T^2}\right). \quad (7.12)$$

We used the standard values of the parameters $a = 1.0$, $b = 0.7$ GeV$^2$ and $\sigma_T = 0.4$ GeV. These values were known to reproduce the data distribution of the reaction $e^+e^- \rightarrow hadrons$ at PEP and PETRA energies ($\sqrt{s} \sim 30$ GeV), quite well.
7.1.2 Detector Simulation

The generated $e^+e^- \rightarrow e^+e^- + \text{hadrons}$ events are passed through the TOPAZ simulator. The TOPAZ simulator consists of various program packages which simulate the response of each detector. The generated particles are swum through the detectors taking into account the effects of multiple scattering, ionizations in the chamber's gas, energy loss, electromagnetic showers and decay in the detector. Electromagnetic shower and nuclear interactions are simulated using the Electron Gamma Shower (EGS) code [37] and the GHEISHA code [44], respectively.

7.1.3 Trigger simulation

In the experimental data, as described in Sec. 3.4, the events which have large energy deposits in the ECL and BCL are identified by the Energy Trigger. On the other hand, the events which have enough charged tracks in the TPC are identified by the Charged Trigger. The Charged Trigger has two steps. The first step is called the "Pre-trigger", and the next step is called "TPC Trigger". The Pre-trigger opens the TPC Trigger.

ECL events and BCL events are identified by the Energy Trigger. The Energy Trigger condition is loose compared to the event selection criteria. On the other hand, FCL events are identified by the Charged Trigger and are sensitive to the detailed condition of the Charged Pre-trigger. Therefore the trigger conditions were simulated using a Trigger simulator for FCL events.

The Charged Pre-trigger requires that there must be at least two back-to-back tracks in the Trigger Chamber (TCH) as shown in Fig.7.3(a). Each tracks in the TCH should have corresponding hits in the Time of Flight counter (TOF) as shown in Fig.7.3(b). The latter condition determines the minimum transverse momentum ($p_t$) of tracks. The trigger efficiency curve as a function of the transverse momentum $p_t$ is shown in Fig.7.4. Events whose $p_t$ being greater than 350 MeV, 500 MeV and 1 GeV, are always triggered, in 1/32 : 7/32, 1/32 : 5/32 and 1/32 : 3/32 condition, respectively.

Fig. 7.5 shows the principle of the TPC Trigger. The TPC Trigger requires at least two tracks in the TPC.

The Charged Trigger condition was changed according to the beam condition and those changes are summarized in Table 7.1. The Monte Carlo events are
Figure 7.3: The principle of the Charged Pre-trigger. (a) shows a trigger condition for two tracks of back-to-back condition, and (b) shows for each tracks of $p_t$ condition.
Figure 7.4: The trigger efficiency in the cosmic ray test.

Figure 7.5: The principle of the TPC trigger.
<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>Integrated Luminosity $(pb^{-1})$</th>
<th>$P_{4}$</th>
<th>Back to Back</th>
<th>TPC 2nd level</th>
<th>Software</th>
</tr>
</thead>
<tbody>
<tr>
<td>58.0</td>
<td>1.64 ± 0.03 ± 0.07</td>
<td>1-3</td>
<td>1-3</td>
<td>/8-5/8</td>
<td></td>
</tr>
<tr>
<td>58.0</td>
<td>0.39 ± 0.01 ± 0.02</td>
<td>1-3</td>
<td>1-7</td>
<td>/8-5/8</td>
<td></td>
</tr>
<tr>
<td>58.0</td>
<td>2.02 ± 0.03 ± 0.08</td>
<td>1-3</td>
<td>1-3</td>
<td>/8-5/8</td>
<td></td>
</tr>
<tr>
<td>58.0</td>
<td>0.34 ± 0.01 ± 0.01</td>
<td>1-3</td>
<td>1-7</td>
<td>/8-5/8</td>
<td></td>
</tr>
<tr>
<td>58.0</td>
<td>4.61 ± 0.05 ± 0.18</td>
<td>1-7</td>
<td>1-9</td>
<td>/8-5/8</td>
<td></td>
</tr>
<tr>
<td>58.0</td>
<td>12.55 ± 0.08 ± 0.50</td>
<td></td>
<td></td>
<td></td>
<td>on</td>
</tr>
<tr>
<td>58.0</td>
<td>8.81 ± 0.06 ± 0.35</td>
<td>1-7</td>
<td>1-9</td>
<td>/8-5/8</td>
<td></td>
</tr>
<tr>
<td>58.0</td>
<td>20.29 ± 0.10 ± 0.81</td>
<td>1-7</td>
<td>1-9</td>
<td>/8-5/8</td>
<td></td>
</tr>
<tr>
<td>58.0</td>
<td>3.84 ± 0.04 ± 0.15</td>
<td></td>
<td></td>
<td></td>
<td>on</td>
</tr>
<tr>
<td>58.0</td>
<td>34.82 ± 0.13 ± 1.39</td>
<td>1-7</td>
<td>1-5</td>
<td>/8-3/8</td>
<td>on</td>
</tr>
</tbody>
</table>

Table 7.1: Trigger condition.

simulated for each trigger condition.

### 7.1.4 Results of Monte Carlo Simulation

The simulated events are then passed the same reconstruction program as the experimental data. The reconstructed Monte Carlo events are selected using the same selection criteria as that for real data.

The expected numbers of events from FKP$(u,d)$+QPM$(c,b)$+VMD with $\Lambda = 0.2$ GeV and $p_{T} = 0.5$ GeV are $1672.1 \pm 23.2 \pm 66.9$, $189.3 \pm 6.6 \pm 7.6$ and $14.3 \pm 0.4 \pm 0.6$ events for FCL events, ECL events and BCL events, respectively after normalizing to the corresponding luminosity of the real data. The errors listed are the statistical error and the systematic error, respectively. The systematic error is estimated from the error in the luminosity measurement: 4%.

### 7.2 Background estimation

The possible contamination of background in the final sample is studied using the Monte Carlo simulation as described in the previous section. The background events are also passed through the the TOPAZ simulator and the trigger simulator in the same way as the single-tag events.
1) \( e^+e^- \rightarrow e^+e^-\tau^+\tau^- \)

The differential cross section for the process depicted in Fig.6.3 is calculated by Kuroda [7]. The differential cross section is integrated using BASES and the events are generated from the BASES's results.

The contamination of the single-tag sample due to this process estimated to be 84 ± 1.9 ± 3.4 (4.2 %), 19.6 ± 0.6 ± 0.8 (10.4 %) and 2.0 ± 0.3 ± 0.1 (6.2 %) events for FCL events, ECL events and BCL events, respectively.

2) Inelastic Compton scattering

The differential cross section for inelastic Compton scattering is also calculated using Kuroda’s program. Produced quarks were fragmented using the LUND (version 6.3) program using the default values of the parameters.

The contaminations due to this process are estimated to be 36.7 ± 1.5 ± 1.5 (1.8 %), 17.5 ± 1.2 ± 0.7 (9.2 %) and 9.4 ± 0.6 ± 0.4 (29.4 %) events for FCL, ECL and BCL events, respectively.

3) \( e^+e^- \rightarrow q \bar{q}, q \gamma \)

We used the LUND Monte Carlo (version 6.3) program to generate hadronic events in the one photon process \( e^+e^- \rightarrow \text{hadrons} \). This generator includes electro-weak effects and initial radiation up to \( O(a^3) \) as depicted in Fig.6.5.

We studied several properties of hadronic events after applying rough cuts (cuts 1 and 2 in Section ??). Fig. 7.6 shows the scatter plots of the longitudinal momentum distribution versus the visible energy of hadron systems. (a) shows the one photon-process, (b) shows \( e^+e^-q \bar{q} \) events and (c) is the experimental data distribution of two-photon events. The longitudinal momentum imbalances of hadron events in the one-photon process are smaller than those of the \( e^+e^-q \bar{q} \) events. We can remove the background of hadronic events in one-photon process, by applying a longitudinal imbalance cut. In Fig. 7.7 (a), the opening angle distributions between a tagged \( e^\pm \) and the other particles from hadronic events in the one-photon process, \( e^+e^-q \bar{q} \) and experimental data are shown. In hadron events in the one-photon process, there is a sharp peak at 0.2 radian. When a tagged electron is isolated, this peak should not be seen. In \( e^+e^-q \bar{q} \) events, this peak does not exist. In order to select events with
Figure 7.6: The scatter plot of the longitudinal momentum imbalance vs. the visible energy of the hadron system. The distributions of simulated hadronic events in the one-photon process and $e^+e^-q\bar{q}$ events are shown in (a) and (b), respectively. That of experimental data are shown in (c).
Figure 7.7: The opening angle distributions between a tagged $e^\pm$ and the other particles of hadronic events in the one-photon process, $e^+e^-q\bar{q}$ events and data are shown in (a), (b) and (c). The momentum summing distribution within 0.1 radian around tagging $e^\pm$ of hadronic events, $e^+e^-q\bar{q}$ events and data are shown in (d), (e) and (f).
an isolated tagged electron, the momenta within 0.4 radian around tagged $e^\pm$
are summed. The distribution of this quantity is shown in Fig. 7.7 (b). We
can remove hadronic events from one-photon processes, effectively by selecting
the events which have small summed momenta around the tagged electron.

After applying all cuts, the contaminations are estimated to be $27.6 \pm 6.1 \pm$
1.1 (13.3 % ), $18.5 \pm 1.8 \pm 0.8$ (9.8 % ) and $3.2 \pm 0.7 \pm 0.1$ (8.1 % ) events
for FCL events, ECL events and BCL events, respectively.

4) $e^+e^- \rightarrow \tau^+\tau^-, \tau^+\tau^-\gamma$

The contamination to single-tag events are calculated in electroweak theory up
to $O(q^3)$ as depicted in Fig.6.5. The distribution of the sum of the transverse
momentum vectors of $\tau^+\tau^-$ events are shown in Fig. 7.8 together with that of
$e^+e^-q\bar{q}$ events, hadronic events via the one photon process and experimental
data. We can remove $\tau^+\tau^-$ events by selecting events with balancing transverse
momenta. After supplying all cuts, the contaminations are estimated to
be $0.6 \pm 0.2 \pm 0.2$ and and $0.2 \pm 0.1 \pm 0.01$ for ECL events and BCL events,
respectively. This background is negligibly small.

5) Beam-gas interactions

The background events from beam-gas interactions are estimated from the
event vertex $V_z$ distribution. Fig.7.9 shows the $V_z$ distribution of ECL events
with the $e^+e^-q\bar{q}$ events of Monte Carlo simulation. As shown in Fig.7.9, the
event vertex has a sharp peak at $z = 0$. These events come from the beam-
beam interaction. The $z$ distribution of beam-gas interactions must show a
flat $V_z$ distribution. We can not see the excess of beam-gas interaction events
in the ECL events. The background events from beam-gas interactions are
negligibly small in ECL events and BCL events.

In FCL events, this background can not be neglected, because the FCL is
located near the beam pipe so that the FCL events are influenced by beam
conditions. The vertex $z$ distribution for FCL events is shown in Fig. 7.10 .
In Fig. 7.10, we can see a clear signal on top of a roughly flat distribution
which comes from beam-gas interactions. We assumed a flat level for the
beam-gas background to estimate the contamination. The beam-gas events
are subtracted from the data by assigning a negative weight to the events in
Figure 7.8: The distribution of the sum of transverse momentum vectors. The distributions of simulated $\tau^+\tau^-$, hadronic events in the one-photon process and $e^+e^-q\bar{q}$ events are shown in (a), (b) and (c), respectively. That of ECL events are shown in (d).
Figure 7.9: The ECL event distribution of event vertex $z$ are shown with that of the $e^+e^-q\bar{q}$ events (solid line) of Monte Carlo simulation.

the side bands of the vertex $z$ distribution. The contamination is $140 \pm 12$

7.3 Experimental results

The above mentioned backgrounds will be subtracted from the selected events on a bin by bin basis in a future analysis. After subtracting the background, 1708.7 ± 47.0 ± 5.9 events, 174.8 ± 15.2 ± 2.2 events and 17.2 ± 6.1 ± 0.6 events remain as signal events for FCL, ECL and BCL events, respectively. The results are summarized with the Monte Carlo predictions of the FKP model with $\Lambda = 0.2$ GeV and $p_t^0 = 0.5$ GeV in Table.7.2. The number of signal events agrees with that of the the Monte Carlo predictions within errors. The distributions for the experimental data are shown in Fig.7.11 (a) ~ (l). These results are discussed in the next section.
Figure 7.10: The FCL event distribution of event vertex $z$ distribution.

<table>
<thead>
<tr>
<th>Tagging device</th>
<th>FCL</th>
<th>ECL</th>
<th>BCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; Q^2 &gt;$ (GeV$^2$)</td>
<td>9.7</td>
<td>78.</td>
<td>320.</td>
</tr>
<tr>
<td>$&lt; \sqrt{s} &gt;$ (GeV)</td>
<td>58.0</td>
<td>57.9</td>
<td>57.9</td>
</tr>
<tr>
<td>Luminosity (pb$^{-1}$)</td>
<td>89.3 ± 3.6</td>
<td>113.9 ± 4.6</td>
<td>113.9 ± 4.6</td>
</tr>
<tr>
<td>Observed events (events)</td>
<td>1997 ± 45</td>
<td>231 ± 15</td>
<td>32 ± 6</td>
</tr>
<tr>
<td><strong>Background events (events)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e^+e^- \rightarrow e^+e^-\tau^+\tau^-$</td>
<td>84 ± 1.9 ± 3.4</td>
<td>19.6 ± 0.6 ± 0.8</td>
<td>2.0 ± 0.3 ± 0.1</td>
</tr>
<tr>
<td>Inelastic Compton</td>
<td>36.7 ± 1.5 ± 1.5</td>
<td>17.5 ± 1.2 ± 0.7</td>
<td>9.4 ± 0.6 ± 0.4</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow q\bar{q}, q\bar{q}\gamma$</td>
<td>27.6 ± 6.1 ± 1.1</td>
<td>18.5 ± 1.8 ± 0.8</td>
<td>3.2 ± 0.7 ± 0.1</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \tau^+\tau^-, \tau^+\tau^-\gamma$</td>
<td>negligible</td>
<td>0.6 ± 0.2 ± 0.02</td>
<td>0.2 ± 0.1 ± 0.1</td>
</tr>
<tr>
<td>Beam gas</td>
<td>140 ± 12</td>
<td>negligible</td>
<td>negligible</td>
</tr>
<tr>
<td><strong>Signal events (events)</strong></td>
<td>1708.7 ± 47.0 ± 5.9</td>
<td>174.8 ± 15.2 ± 2.2</td>
<td>17.2 ± 6.1 ± 0.6</td>
</tr>
<tr>
<td><strong>M.C. prediction (events)</strong></td>
<td>1672.1 ± 23.2 ± 66.9</td>
<td>189.3 ± 6.6 ± 7.6</td>
<td>14.3 ± 0.4 ± 0.6</td>
</tr>
</tbody>
</table>

Table 7.2: The summary of the experimental results with the those of background estimation and the Monte Carlo prediction.
7.4 Comparison with theoretical predictions

In this section, the observed distributions of data for ECL events are compared with two kinds of theoretical models. A more detailed discussion will be given in Chapter 9, where acceptance corrected data will be compared with various theoretical calculations.

As described in Section 7.1, Monte Carlo events are generated according to the differential cross section (Eq. (7.2)). For light quarks (u, d and s), we used two kinds of models for the photon structure function $F_2^p$. One is the FKP model (Eq. (2.38)) and the other is the QPM (Eq. (2.23)). The hadron-like part of the photon structure function is described by the VMD model. The heavy quark (c and b) contributions are generated using QPM (Eq. (2.24)). The generated events were simulated and passed through the same analysis program as the experimental data.

Various distributions of experimental data are compared with the Monte Carlo predictions as shown in Fig. 7.11 (a) ~ (l). The solid circles in the figures are the experimental data after subtracting backgrounds. The solid line histogram is the theoretical prediction of FKP(u, d, s) + QPM(c, b) + VMD, and the dashed line is that of QPM(u, d, s) + QPM(c, b) + VMD.

Distributions of the energy and scattering angle for tagged electrons or positrons are shown in (a) and (b), respectively. The charged multiplicity distribution and angular distribution of charged tracks are shown in (c) and (d), respectively. The transverse momentum distribution for each track is shown in (e). Distributions of multiplicity and energy in ECL and BCL are shown in (f), (g), (h) and (i), respectively. The square of the momentum transfer $Q^2$ of the virtual photon and the visible mass of hadron system $W_{vis}$ are shown in (j) and (k), respectively. The visible scaling variable $x_{vis}$ is calculated from $Q^2$ and $W_{vis}$ as follows:

$$x_{vis} = \frac{Q^2}{Q^2 + W_{vis}^2}.$$  \hspace{1cm} (7.13)

The $x_{vis}$ distribution is shown in (l). Predictions from models are in good agreement with the experimental data within error, in all distributions.
Figure 7.11: (a)–(e) Comparison of the visible experimental data with two kinds of theoretical predictions. One is \( \text{FKP}(u, d, s) + \text{QPM}(c, b) + \text{VMD} \) with cut-off \( p_T^c = 0.5 \text{ GeV/c} \) and QCD scale parameter \( \Lambda = 0.2 \text{ GeV} \) (shown with the solid line), and the other is \( \text{QPM}(u, d, s) + \text{QPM}(c, b) + \text{VMD} \) (shown with a dashed line histogram).
Fig. 7.11: (f)–(k) Comparison of the visible experimental data with two kinds of theoretical predictions. One is FKP($u, d, s$) + QPM($c, b$) + VMD with cutoff $p_t^0 = 0.5$ GeV/$c$ and QCD scale parameter $\Lambda = 0.2$ GeV (shown with the solid line), and the other is QPM($u, d, s$) + QPM($c, b$) + VMD (shown with
a dashed line histogram ).
Fig. 7.11: (1) Comparison of the visible experimental data with two kinds of theoretical predictions. One is FKP\((u, d, s) + \text{QPM}(c, b) + \text{VMD}\) with cut-off \(p_T^0 = 0.5 \text{ GeV/c}\) and QCD scale parameter \(\Lambda = 0.2 \text{ GeV}\) (shown with the solid line), and the other is \(\text{QPM}(u, d, s) + \text{QPM}(c, b) + \text{VMD}\) (shown with a dashed line histogram).

So far we have compared the data with the theoretical predictions in the visible distributions. However, if one wants to compare another model with our experimental data, he or she must know the detailed structure of our detector. It is very convenient that the data should be presented after correcting for detector effects. However, it is not easy to make an acceptance correction for two-photon events since only some of the hadrons are observed in all of the detectors used in \(\text{e}^+\text{e}^-\) experiments. Therefore, a special technique of acceptance correction is necessary to correct the detector effects in two-photon events. In the next Chapter this technique is discussed in detail.
Chapter 8

Acceptance Correction

In general, detectors have limited acceptance and finite resolution. These are different for each detector. In order to compare our data with other group’s measurements or to compare theoretical predictions for some physical parameters, we have to correct for detector effects to obtain the original distribution.

The acceptance correction for single-tag events in the two-photon process is not easy. Since the produced hadrons are boosted into the direction of the center of mass system of the photon-photon collisions, they are scattered at small scattering angles with respect to the beam axis in the laboratory system. Therefore, we cannot detect all of the produced hadrons and the visible hadron mass \( W_{vis} \) becomes smaller than the true value \( W_{true} \). The momentum fraction \( x \) of quarks in the target photon is calculated to be \( \frac{Q^2}{Q^2+m^2} \). Because \( W_{vis} \) is smaller than \( W_{true} \), the visible momentum fraction \( x_{vis} \) becomes larger than a true momentum fraction \( x_{true} \) in usual cases.

In general, if the \( x_{true} \) distribution \( f(x_{true}) \) is defined over the range \( a \leq x_{true} \leq b \), the visible distribution \( g(x_{vis}) \) is obtained by the following integral:

\[
g(x_{vis}) = \int_a^b A(x_{vis}, x_{true}) f(x_{true}) dx_{true} \, ,
\]

where \( A(x_{vis}, x_{true}) \) describes the response of the detector including the smearing effects of \( x_{vis} \). An accurate determination of the response function \( A(x_{vis}, x_{true}) \) is very important.

The function \( A(x_{vis}, x_{true}) \) can be determined by using Monte Carlo simulation. From \( A(x_{vis}, x_{true}) \) and the experimental visible distribution \( g(x_{vis}) \), the true distribution \( f(x_{true}) \) will be reconstructed.
The method to reconstruct \( f(x_{\text{true}}) \) is usually called “unfolding” method. We have used the unfolding method developed by Blobel \cite{47}, which is the most common method used in the measurement of the photon structure function. In this Chapter, we will review Blobel’s unfolding method. The comments on other unfolding methods used commonly are given in Appendix A, briefly.

### 8.1 Unfolding method by V. Blobel

In Blobel’s unfolding method, the true distribution \( f(x_{\text{true}}) \) is expanded with a set of orthogonal functions \( (p_j(x_{\text{true}})) \):

\[
f(x_{\text{true}}) = \sum_{j=1}^{m} a_j p_j(x_{\text{true}}),
\]

where \( a_j \) is a coefficient of this basic function. The cubic B-spline function is used as the basic function \( p_j(x_{\text{true}}) \). The cubic B-spline function is a set of four cubic polynomials \(^1\). These polynomials are joined together in such a way that the function is continuous up to the second derivative at the junctions. These junctions are called knots. A merit of the B-spline function is the spurious statistical fluctuations in the solution \( f(x_{\text{true}}) \) can be removed easily. For this purpose Blobel introduces a cut off \( m_0 \) in \( j \), which is called the regularization method. The term with \( j > m_0 \) is neglected, since these higher order terms in \( f(x_{\text{true}}) \) come from fluctuations. The determination of \( m_0 \) is very important, in order to avoid any possible bias coming from the statistical errors in Monte Carlo events.

Blobel’s unfolding method consists of three steps. In the first step, the \( g(x_{\text{vis}}) \) and \( f(x_{\text{true}}) \) in Eq. (8.1) and Eq. (8.2) are replaced with the number in the \( i \)’th bin ( i.e. \( g_i, f_k \) ). In the second step, the \( A_{ij} \) or the coefficient \( a_j \) in Eq. (8.2) is determined using Monte Carlo events. This solution is called the non-regularized solution. In the third step, the solution with regularization is evaluated using the solution without regularization. In the next subsection, details of each step will be mentioned.

\(^1\) This spline function is commonly used in the determination of the smooth curves for given points. This function is very convenient to make “smoothing” without introducing any kind of bias in the smoothing step.
8.1.1 Discretization

In the first step, the discretization of Eq. (8.1) is done in two steps. At first, the function $f(x_{true})$ is parametrized by a sum of basic functions $p_j(x_{true})$

$$f(x_{true}) = \sum_{j=1}^{m} a_j p_j(x_{true}) . \quad (8.3)$$

Using this parameterization, Eq. (8.1) is rewritten as follows:

$$g(x_{vis}) = \int_a^b A(x_{vis}, x_{true}) f(x_{true}) dx_{true}$$

$$= \sum_{j=1}^{m} a_j \left[ \int_a^b A(x_{vis}, x_{true}) p_j(x_{true}) dx_{true} \right]$$

$$= \sum_{j=1}^{m} a_j A_j(x_{vis})$$

with

$$A_j(x_{vis}) = \int_a^b A(x_{vis}, x_{true}) p_j(x_{true}) dx_{true} . \quad (8.4)$$

In the next step, Eq. (8.4) is represented by all $x_{vis}$-dependent functions assuming a certain set of bin-limits $x_{vis}^0, x_{vis}^1, \ldots, x_{vis}^n$:

$$g_i = \int_{x_{vis}^{i-1}}^{x_{vis}^i} g(x_{vis}) dx_{vis} = \sum_j a_j A_{ij} , \text{ where } A_{ij} = \int_{x_{vis}^{i-1}}^{x_{vis}^i} A_j(x_{vis}) dx_{vis} . \quad (8.5)$$

The elements $A_{ij}$ can be defined by Monte Carlo events.

The choice of the basic function $p_j$ is also important. The simplest choice of $p_j$ is:

$$p_j(x_{true}) = \begin{cases} 1 & \text{for } t_{j-1} \leq x_{true} < t_j \\ 0 & \text{for otherwise} \end{cases}$$

with

$$\sum_{j=1}^{m} p_j(x_{true}) = 1$$

with a set of knots $t_0, t_1, t_2, \ldots, t_m$. However in order to get a smooth curve solution of $f(x_{true})$, cubic B-splines are much better than this simple function. In this analysis, we used the cubic B-spline functions as $p_j$.

If we get the coefficient $a_j$, we can know the solution $f(x_{true})$. In the next subsection we will review how to determine the coefficient $a_j$. 
8.1.2 Unfolding without Regularization

The true distribution of Monte Carlo events and experimental data are represented by $f(x_{true})^{M.C.}$ and $f(x_{true})^{Data}$, respectively. These distributions are expanded with a set of cubic B-spline functions, which are orthogonal polynomials, as follows:

$$f(x_{true})^{M.C.} = \sum_j a_j^{M.C.} p_j(x_{true}) ,$$

(8.7)

$$f(x_{true})^{Data} = \sum_j a_j^{Data} p_j(x_{true}) ,$$

(8.8)

and the coefficients $a_j$ can be calculated by the following summations using a discrete set of points $(x_{vis}^i, x_{true}^i)$, $i = 1 \ldots n$,

$$a_j^{M.C.} = \sum_{i=1}^n w_i p_j(x_{true}^i) x_{vis}^i M.C. ,$$

(8.9)

$$a_j^{Data} = \sum_{i=1}^n w_i p_j(x_{true}^i) x_{vis}^i Data ,$$

(8.10)

where $x_{true}^i$ and $x_{vis}^i$ are the true and visible data points, respectively and $w_i$ is the weight of an individual data point.

To obtain $a_j^{Data}$, the maximum likelihood method is used. The minimum of the negative logarithm of the likelihood function is given:

$$S(a^{Data}) = -\sum_{i=1}^n \ln P(f_i^{Data} | f_i^{M.C.}) ,$$

(8.11)

where $P(f_i^{Data} | f_i^{M.C.})$ is the probability of observing $f_i^{Data}$ for the mean value of $f_i^{M.C.}$. Introducing the Gaussian density for $P(f_i^{Data} | f_i^{M.C.})$, $S(a^{Data})$ becomes

$$S(a^{Data}) = \frac{1}{2} \sum_{i=1}^n \frac{(f_i^{Data} - f_i^{M.C.})^2}{\sigma_i^2} ,$$

(8.12)

$$\sigma_i^2 = f_i^{Data} .$$

(8.13)

The $S(a^{Data})$ can be written using matrix notation as follows:

$$S(a^{Data}) = S(\hat{a}) - (a^{Data} - \hat{a})^T h + \frac{1}{2} (a^{Data} - \hat{a})^T H (a^{Data} - \hat{a}) ,$$

(8.14)

$$h_j = -\frac{\partial S}{\partial a_j^{Data}} , \quad H_{jk} = -\frac{\partial^2 S}{\partial a_j^{Data} \partial a_k^{Data}} ,$$

(8.15)

where $\hat{a}$ is an approximate solution. The coefficients $a^{Data}$ are determined so as to satisfy the minimum condition $\nabla S = 0$, which is the solution without regularization.
8.1.3 Unfolding with Regularization

The following regularization parameter \( \tau \) and the smoothness \( r(a^{Data}) \) are introduced to obtain the regularized solutions:

\[
R(a^{Data}) = S(a^{Data}) + \frac{1}{2} \tau r(a^{Data}) ,
\]

\[
r(a^{Data}) = \int |f''(x_{true})|^2 dx_{true} .
\]

And the coefficients \( a^{Data} \) are determined to satisfy \( \nabla R(a^{Data}) = 0 \). Because \( f(x_{true}) \) are parametrized by a sum of cubic B-splines, the smoothness \( r(a^{Data}) \) can be represented using a matrix \( C \) as follows:

\[
r(a^{Data}) = a^{Data}^T C a^{Data} .
\]

To get the solution, at first, the matrix \( H \) described in Eq. (8.15) is transformed to a diagonal matrix \( D \),

\[
D = U_1^T H U_1 ,
\]

\[
U_1^T U_1 = I_{mm} ,
\]

The eigenvalues \( D_{jj} \) are arranged as follows:

\[
D_{11} \geq D_{22} \geq D_{33} \cdots \geq D_{mm} .
\]

A transformation is defined between the parameter vector \( a^{Data} \) and another vector \( a_1 \) by

\[
a^{Data} = U_1 D^{-\frac{1}{2}} a_1 ,
\]

where \( D^{\frac{1}{2}} \) satisfies

\[
D^{\frac{1}{2}} D^{\frac{1}{2}} = D .
\]

The Eq. (8.16) is rewritten by using the transformation Eq. (8.22) from \( a^{Data} \) to \( a_1 \),

\[
R(a_1) = -a_1^T D^{-\frac{1}{2}} U_1^T (H \dot{a} + h) + \frac{1}{2} a_1^T a_1 + \frac{1}{2} \tau a_1^T C_1 a_1 ,
\]

with

\[
C_1 = D^{-\frac{1}{2}} U_1^T C U_1 D^{-\frac{1}{2}} .
\]

Next, the matrix \( C_1 \) is transformed to a diagonal matrix \( G \),

\[
G = U_2^T C_1 U_2 ,
\]

with

\[
U_2^T U_2 = I_{mm} ,
\]

\[
G_{11} \leq G_{22} \leq \cdots \leq G_{mm} .
\]
The additional transformation is defined by

$$a_1 = U_2 d' , \quad (8.29)$$

where $a_1$ satisfies $a_1^T a_1 = a'^T d'$. The function to be minimized becomes

$$R(d') = -a'^T U_2^T D^{-\frac{1}{2}} U_1^T (H \dot{a} + h) + \frac{1}{2} a'^T (I + \tau G) a' . \quad (8.30)$$

The regularized solution, denoted by a hat, is calculated from the minimum condition $\nabla R = 0$:

$$\dot{d}' = (I + \tau G)^{-1} U_2^T D^{-\frac{1}{2}} U_1^T (H \ddot{a} + h) , \quad (8.31)$$

whereas the unregularized solution, denoted by a bar, is given when $\tau = 0$, as follows:

$$\bar{d}' = U_2^T D^{-\frac{1}{2}} U_1^T (H \dot{a} + h) . \quad (8.32)$$

This result can be transformed back to $\bar{a}$ using Eq. (8.22), (8.29),

$$\bar{a} = U_1 D^{-\frac{1}{2}} U_2 \bar{d}' , \quad (8.33)$$

then

$$\bar{f}(x_{true}) = \sum_{j=1}^{m} \bar{a}_j p_j(x_{true}) \quad (8.34)$$

$$= \sum_{j=1}^{m} \bar{a}_j p_j(x_{true}) . \quad (8.35)$$

In this parametrization the curvature Eq. (8.17) is given by

$$r(\bar{a}) = \int [\bar{f}'(x_{true})]^2 dx \quad (8.36)$$

$$= \bar{a}^T C \bar{a} \quad (8.37)$$

$$= \sum_{j=1}^{m} (\bar{a}_j')^2 G_{jj} . \quad (8.38)$$

**Determination of the regularization parameter**

The coefficients of the regularization solution are related to that of the unregularized solution, from Eq.(8.31) and (8.32), as follows:

$$\dot{a}'_j = \frac{1}{1 + \tau G_{jj}} \bar{a}'_j . \quad (8.39)$$
As seen in Eq. (8.39), the coefficients of the regularized solution can be obtained by multiplying that of the unregularized solution by a factor. If $\tau G_{jj} \ll 1$, the factor is close to 1 and if $\tau G_{jj} \gg 1$, the factor is almost 0. Thus the regularization means a smooth cut-off. The sum of all factors can be considered as an effective number $m_0$. For a given $m_0$ the regularization parameter $\tau$ can be defined by

$$m_0 = \sum_{j=1}^{m} \frac{1}{1 + \tau G_{jj}} .$$

(8.40)

**Results and optimum binning**

After fixing the regularization parameter $\tau$, the regularized solution can be calculated using Eq. (8.39), and the covariance matrix of $\vec{\alpha}$ is given by,

$$V(\vec{\alpha}) = (I + \tau G)^{-2} .$$

(8.41)

Of course these results can be transformed back using Eq. (8.22) and (8.29).

Finally the resulting coefficients are converted to a set of $m_0$ data points $f_k$ by integrating $f(x_{true})$ over small region of $x$ as follows:

$$f_k = \left( \int_{x_{true}^{k-1}}^{x_{true}^{k}} f(x_{true}) dx_{true} \right) / (x_{true}^{k} - x_{true}^{k-1})
= \left( \sum_{j=1}^{m} a_j \int_{x_{true}^{k-1}}^{x_{true}^{k}} p_j(x_{true}) dx_{true} \right) / (x_{true}^{k} - x_{true}^{k-1}) .$$

(8.42)

Here $f_k$ is the average value of the function $f(x_{true})$ in $[x_{true}^{k-1}, x_{true}^{k}]$, and is a linear function of the coefficient $a_j$. Therefore we can calculate the errors straightforwardly.

The integration region should be chosen so that the correlations between the data points should become as small as possible. To suppress higher order effects, the optimal bin limits can be chosen at the extreme values of $p_{m_0+1}$. The function $p_{m_0+1}$ has just $m_0$ zero points and $(m_0-1)$ extreme values. If we choose these values for the bin limits, the contribution of $p_{m_0+1}$ is cancelled as shown in Fig.8.1.

Fig.8.2(a) shows the generated data with the original function ( solid line ). The generated data are then simulated as follows. The acceptance probability is assumed to be

$$P_{acc}(x) = 1 - \frac{1}{2}(x - 1)^2 ,$$

(8.43)
Figure 8.1: Examples of the normalized orthogonal functions at \( m_0 = 12 \). (a) shows the normalized orthogonal functions used to represent the solution, and (b) shows the \( p_{m_0+1} \) contribution.
Figure 8.2: Histogram of the generated data and unfolding results. (a) shows the generated data with original function and (b) shows the simulated data of (a) with original function. (c) shows the unfolding results with original function.
and the true values $x$ are transformed to a variable $y_{tr}$ by the function

$$y_{tr} = x - 0.2 \frac{x^2}{4}.$$  \hfill (8.44)

Then $y_{tr}$ is assumed to be measured with a resolution $\sigma = 0.1$. The result of the simulation is shown in (b) with the original function (solid line). The unfolding result with regularization is compared in (c) with the original function. The results reproduced the original function quite well.
Chapter 9

Results and Discussion

In this Chapter, we first show the results of the unfolded $F_2^\gamma$ distribution. Then, we will compare the results with theoretical predictions. Finally, the $Q^2$-dependence of the averaged photon structure function in the medium $x$ region are shown and compared with other data from PETRA, PEP and TRISTAN(AMY).

9.1 Experimental results

Using Blobel’s unfolding technique, we have extracted the photon structure function $F_2^\gamma$ using the FCL and ECL event $x_{vis}$ and $Q^2$ distributions. The $Q^2$ value of the FCL events ranges from 7 to 30 $(\text{GeV}/c)^2$ with the average being 9.7 $(\text{GeV}/c)^2$, while the $Q^2$ of the ECL events ranges from 45 to 200 $(\text{GeV}/c)^2$ with the average value of 87 $(\text{GeV}/c)^2$. In Table 9.1, we list the measured values of $F_2^\gamma$ at $<Q^2>=9.7$ and 87 $(\text{GeV}/c)^2$, where the first errors are statistical and the second systematic.

The measured structure functions $F_2^\gamma$ at $<Q^2>=9.7$ and 87 GeV/$c^2$ are shown in Fig.7.11 (a) and (b), respectively, with the quadratic sum of the statistical and systematic errors. The systematic errors in the measured $F_2^\gamma$ include following sources.

1. A 3% overall normalization error is due to the uncertainty of the trigger efficiency for FCL events.

2. A 4% overall normalization error is associated with the uncertainty in the luminosity measurement.
\[
\langle Q^2 \rangle = 9.7 \text{ (GeV/c)}^2 \text{ (} 7 \sim 30 \text{ (GeV/c)}^2 \text{)}
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \langle x \rangle )</th>
<th>( F_2^\gamma / \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02 \sim 0.30</td>
<td>0.16</td>
<td>0.482 \pm 0.092 \pm</td>
</tr>
<tr>
<td>0.30 \sim 0.58</td>
<td>0.43</td>
<td>0.463 \pm 0.094 \pm</td>
</tr>
<tr>
<td>0.58 \sim 0.85</td>
<td>0.71</td>
<td>0.389 \pm 0.077 \pm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \langle x \rangle )</th>
<th>( F_2^\gamma / \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03 \sim 0.32</td>
<td>0.18</td>
<td>0.563 \pm 0.296 \pm</td>
</tr>
<tr>
<td>0.32 \sim 0.60</td>
<td>0.46</td>
<td>0.525 \pm 0.285 \pm</td>
</tr>
<tr>
<td>0.60 \sim 0.99</td>
<td>0.79</td>
<td>0.638 \pm 0.262 \pm</td>
</tr>
</tbody>
</table>

Table 9.1: The measured value of the photon structure function \( F_2^\gamma / \alpha \) at \( \langle Q^2 \rangle = 9.7 \text{ and } 87 \text{ GeV/c}^2 \).

3. Additional systematic errors for \( F_2^\gamma \) may come from the uncertainty in the VMD contributions. We, however, do not include these in the systematic errors.

The total systematic errors are estimated to be 5% by taking the quadratic sums of the individual errors.

By comparing the measurement of \( F_2^\gamma \) at different \( Q^2 \) regions in Fig.7.11 (a) and (b), we can observe clear positive scaling violation at all values of \( x \), i.e., the \( F_2^\gamma \) increase at higher values of \( Q^2 \) in all \( x \) regions. This behavior of the photon structure function is consistent with the in QPM and remains so in the framework of QCD. ( These calculations predict \( F_2^\gamma \propto \ln Q^2 \) behavior. ) This scaling violation of \( F_2^\gamma \) is profoundly different from that observed for the nucleon ( proton or neutron ) structure function. For the nucleon, Bjorken scaling is the signature of the QPM and the logarithmic scaling violation ( \( \sim \ln(\ln Q^2) \) ) is introduced only by QCD.

### 9.2 Comparison with theoretical calculations

In the following subsections, we will compare our measurements of \( F_2^\gamma \) with various theoretical predictions. Although all calculations are based on the
Figure 9.1: Unfolded photon structure function $F_2^\gamma$ distribution at $Q^2 = 9.7$ $(\text{GeV}/c)^2$ (a) and at $Q^2 = 87$ $(\text{GeV}/c)^2$ (b).
QPM or QCD, all predictions contain some ambiguities. These ambiguities come mainly from the treatment of the non-perturbative pieces of $F_2^\gamma$.

### 9.2.1 Comparison with the Quark Parton Model

In Fig. 9.2, the measurements of $F_2^\gamma$ are compared with the theoretical predictions of the Quark Parton Model (QPM). The explicit formulae of the QPM are described in Section 2.2. The solid line in the figure shows the sum of the QPM ($u, d, s, c$) and the VMD contributions. The contributions of VMD and the charm quark are shown by dotted and dashed lines, respectively. The $b$ quark contribution was found to be negligible. In these calculation, the masses of quarks are assumed to be $m_u = m_d = 325$ MeV, $m_s = 500$ MeV, $m_c = 1.6$ GeV and $m_b = 5$ GeV. For the VMD piece, we have used following commonly used form:

$$ (F_2^{\gamma,HAD})_{VMD} = 0.2\alpha(1-x) \quad (9.1) $$

Note, that the charm quark contribution is substantial especially in the high $Q^2$ region of $Q^2 > 78$ (GeV/c)². In the observed $Q^2$ region ($Q^2 > 9.7 \sim 78$ GeV²), the data are reproduced only by the QPM quite well, if the approximate hadronic part (= non-perturbative pieces) is taken into account.

### 9.2.2 Comparison with the QCD parametrization formulae given by DG and LAC

In the DG and LAC approaches, no distinction is made between the point-like and hadron-like contributions to the structure function. They used leading log AP equations to evolve an input parameterization of $F_2^\gamma$ at a given $Q_0^2$ to a different $Q^2$. This is the same procedure taken in the QCD calculations of nucleon structure functions (see Section 2.3.2, for more detail). The data and these parametrizations are compared in Fig. 9.3. The solid circles are the unfolded experimental data. The DG parameterization with $\Lambda = 0.4$ GeV is shown with a solid line. Three kinds of parametrizations are provided by LAC, for different starting values of $Q_0^2$ and different treatments of the gluon distribution. We call them LAC-1 ($Q_0^2 = 4$GeV²), LAC-2 ($Q_0^2 = 4$GeV², different form of gluon distribution) and LAC-3 ($Q_0^2 = 1$GeV²), respectively. The dashed, dotted and dash-dotted lines show the LAC-1, 2 and
Figure 9.2: Comparison of the measured $F_7^g$ with the theoretical prediction of QPM+VMD. The solid line shows the QPM+VMD prediction. The hadronic contribution, given by VMD, and heavy quark contributions are shown by dotted and dashed lines, respectively.
Figure 9.3: Comparison of the measured $F_2^q$ with the theoretical predictions of DG and LAC. The solid line shows the parametrization by DG with $\Lambda = 0.4$ GeV$^2$. LAC’s parametrizations with $\Lambda = 0.2$ GeV$^2$ of Set 1, 2 and 3 are shown in the dashed, dotted and dash-dotted line, respectively. $Q^2_0$’s of Set 1, 2 and 3 are 4, 4 and 1 GeV$^2$, respectively. The gluon distribution is different between Sets 1 and 2.
3 parametrizations respectively. All parametrizations give good descriptions of the data within error.

9.2.3 Comparison with QCD prediction given by FKP

Field, Kapusta and Pogiolli (FKP) introduced a cut-off parameter either \( p_t^0 \) or \( t_c \), in order to separate the point-like (= perturbative) part and hadron-like (= non-perturbative) part of the photon structure function. The point-like part of \( F_2^\gamma \) is calculated in QCD in the region \( p_t > p_t^0 \) (or \( t > t_c \)). The free parameters involved in their calculation are \( p_t^0 \) (or \( t_c \)) and QCD scale parameter of \( \Lambda \). As described in Section 2.3.3, their prediction of \( F_2^\gamma \) is sensitive to the value of \( p_t^0 \) (or \( t_c \)), but not to \( \Lambda \). The data and the FKP predictions are compared in Fig. 9.4. The FKP predictions were calculated with \( \Lambda = 0.2 \) GeV for the values of \( p_t^0 = 0.1, 0.5 \) and 1 GeV/c. These curves include the hadronic contribution given by VMD and heavy quark \( (c \) and \( b) \) contributions. The masses of quarks are taken to be \( m_c = 1.6 \) GeV and \( m_b = 5 \) GeV. FKP’s calculation and the data agrees if \( p_t^0 \) is taken to be less than 1.0 GeV.

9.3 \( Q^2 \)-dependence of the Photon Structure Function

The \( Q^2 \) evolution of the photon structure function is a basic feature of the QPM and QCD. This situation is profoundly different from the nucleon structure function. In the latter case, Bjorken scaling is expected in QPM. In order to study the \( Q^2 \)-dependence of \( F_2^\gamma \), we have measured the average values of \( F_2^\gamma / \alpha \) for the light quarks \( (u, d, s) \) in the intermediate \( x \) region \( 0.3 \sim 0.8 \). The heavy quark contributions \( (\text{mainly the } c \text{ quarks}) \) are subtracted from the data using the formula in Eq. (2.24). For this measurement, we also included the BCL events which cover the \( Q^2 \) range from 300 GeV/c\(^2 \) to 700 GeV/c\(^2 \) with the average value of \( < Q^2 > = 338 \text{ (GeV/c)}^2 \). Our results are

\[
< F_2^\gamma / \alpha > = 0.357 \pm 0.021 \text{ at } < Q^2 > = 3.7 \text{(GeV/c)}^2, \\
< F_2^\gamma / \alpha > = 0.466 \pm 0.063 \text{ at } < Q^2 > = 87 \text{(GeV/c)}^2, \\
< F_2^\gamma / \alpha > = 0.753 \pm 0.363 \text{ at } < Q^2 > = 338 \text{(GeV/c)}^2,
\]
Figure 9.4: Comparison of the measured value of $F_2^\gamma$ with the theoretical predictions by FKP($u, d, s$)+QPM($c, b$)+VMD with $p_T^0 = 0.1$, 0.5 and 1 GeV/c at QCD scale parameter $\Lambda = 0.2$ GeV. The hadronic contribution given by VMD and that of the heavy quark by QPM are shown in the dotted lines.
Figure 9.5: $Q^2$-dependence of the photon structure function $F_2^\gamma$ with previously measured data. The dashed, solid and dotted line shows the theoretical expectation of FKP+VMD with $p_T^0 = 0.1$, 0.5 and 1 GeV/c, respectively. The dash-dotted line shows that of QPM+VMD. The hadronic contribution given by VMD is shown in long-dashed line.
where the error is a quadratic sum of the statistical and systematic errors. These results are shown in Fig. 9.5 together with the measurements of other groups [8]. The dashed, solid and dotted lines show the predictions of FKP with $p_t^0 = 0.1$, 0.5 and 1 GeV/c, respectively, and the dash-dotted line shows the prediction of the QPM. The prediction of FKP with $p_t^0 = 0.1$ GeV is almost equal with that of the QPM. We can see the clear $\ln(Q^2)$ dependence of $F_2^\gamma$ that is expected from the QPM and QCD.
Chapter 10

Conclusion

We have studied hadron production in the two-photon process and measured the photon structure function $F_2^\gamma(x, Q^2)$ at the average $Q^2$ values of 9.7, 87, and 338 (GeV/c)^2. The data were collected by the TOPAZ detector at the TRISTAN $e^+e^-$ collider during the five year period data from 1987 to 1992. The data were taken at the center of mass energies ($\sqrt{s}$) from 52 GeV up to 61.4 GeV, in which about 70% data are taken at the fixed energy of $\sqrt{s} = 58$ GeV. The corresponding luminosities of the data at $Q^2 = 9.7$, 87 and 338 (GeV/c)^2 are 89.3, 113.9 and 113.9 $pb^{-1}$, respectively.

Various distributions for the two-photon events, such as charged multiplicity, $p_t$ of tracks, $Q^2$, mass of the hadron system, and $x$ distributions were compared with the predictions of FKP and QPM using Monte Carlo simulation. The cut-off parameter $p_t^0 = 0.5$ GeV and QCD scale parameter of $\Lambda = 0.2$ GeV were used in the FKP's calculation of $F_2^\gamma$. The Monte Carlo shows good agreement with the data.

Using these Monte Carlo events, the effects of the detector are removed by the unfolding method developed by V. Blobel and we have extracted the photon-structure function $F_2^\gamma$ at the average $Q^2$ values of 9.7 and 87 (GeV)^2. Our data shows clear scale violation behavior which is expected from both the QPM and QCD. $x$-dependence of our data at $<Q^2> = 9.7$ and 87 (GeV)^2 are compared with the QPM prediction and the various QCD predictions of the DG, LAC parametrizations and FKP's calculations. All predictions show good agreement with the experimental data within errors. The cut-off parameter ($p_t^0$) introduced by FKP to separate the point-like and hadron-like parts is found to be less than 1.0 GeV/c. 

142
Finally, we have measured the averaged value of the photon structure function for light quarks \((u,d,s)\) in the intermediate \(x\) region \(0.3 < x < 0.8\). Our results are

\[
\begin{align*}
\langle F_2^a / \alpha \rangle & = 0.357 \pm 0.021 \text{ at } \langle Q^2 \rangle = 9.7 \text{(GeV/c)}^2, \\
\langle F_2^a / \alpha \rangle & = 0.466 \pm 0.063 \text{ at } \langle Q^2 \rangle = 87 \text{(GeV/c)}^2, \\
\langle F_2^a / \alpha \rangle & = 0.753 \pm 0.363 \text{ at } \langle Q^2 \rangle = 338 \text{(GeV/c)}^2,
\end{align*}
\]

where the error is the quadratic sum of the statistical and systematic errors. Our data covers the widest range of \(Q^2\) for a single experiment. The data show the clear positive scaling violation expected by QPM and QCD.
Appendix A

Unfolding methods

There are other unfolding methods than Blobel’s unfolding method described in Section 8.1. In this Appendix, the other methods will be reviewed.

A.1 Correction factor method

If a distribution is to be corrected only for limited detector acceptance, the acceptance correction is not very difficult. In this case we have to consider only the elements $A_{jj}$, the acceptance probability for bin $j$, because all off-diagonal elements $A_{ij}$ are zero. A common method for acceptance correction is as follows. Monte Carlo events with $x$-values are generated according to some assumption $f_{i}^{M.C.}$. Generated events are simulated according to the detector response, and selected using the data analysis routine. The selected events are put into a histogram $g_{i}^{M.C.}$. The bin by bin ratio $g_{i}/f_{i}$ of the histograms gives the value $A_{jj}$ of the acceptance probability for bin $j$. The corrected bin contents $f_{j}^{Data}$ are obtained as follows.

$$f_{j}^{Data} = g_{j}^{Data} \left( \frac{f_{j}^{M.C.}}{g_{j}^{M.C.}} \right).$$  \hspace{1cm} (A.1)

Since $A_{ij} = 0$ for $i \neq j$, the correction factor does not depend on the assumed distribution $f_{i}^{M.C.}$. Next we describe a correction for limited resolution.
A.2 Matrix inversion method

If a correction for limited detector resolution becomes necessary, the situation is completely different. The case with \( n = m \) ( square matrix \( \mathbf{A} \) ) will be considered here. By using a Monte Carlo Simulation, the detector response matrix \( \mathbf{A} \) can be obtained.

\[
\mathbf{g}^{M.C.} = \mathbf{A} \mathbf{f}^{M.C.},
\]

where \( \mathbf{f}^{M.C.} \) is the generated distribution and \( \mathbf{g}^{M.C.} \) is the simulated and selected distribution. The corrected distribution \( \mathbf{f}^{\text{Data}} \) can be obtained as follows,

\[
\mathbf{f}^{\text{Data}} = \mathbf{A}^{-1} \mathbf{g}^{\text{Data}},
\]

where \( \mathbf{g}^{\text{Data}} \) is the distribution measured by the detector and \( \mathbf{A}^{-1} \) is the matrix inversion to \( \mathbf{A} \). This method does not depend on the Monte Carlo distribution \( \mathbf{f}^{M.C.} \). In this method the statistical fluctuations are enhanced and one gets even negative results in some bins.

A.3 Approximate unfolding

Using a Monte Carlo Simulation, a matrix \( C_{ij} \) defined by the following formula can be calculated directly [48],

\[
f_i^{M.C.} = C_{ij} \mathbf{g}^{M.C.}_j,
\]

where \( f^{M.C.} \) is the generated distribution, \( g^{M.C.}_j \) is a simulated distribution deformed for the detector response and \( C_{ij} \) is the transformation matrix between these two distributions. The distribution of experimental data is corrected using the matrix \( C_{ij} \) as follows,

\[
f_i^{\text{Data}} = C_{ij} \mathbf{g}^{\text{Data}}_j.
\]

In this method, the corrected distribution \( f^{\text{Data}} \) is influenced by the the generated distribution \( f^{M.C.} \). Therefore we can use this method if the Monte Carlo model is very close to reality. To get consistent results for different starting distributions, we have to use this method iteratively.

\[
f_i^{M.C., 1\text{st}} = C_{ij} \mathbf{g}^{M.C., 1\text{st}}_j.
\]
\[ f_i^{MC^{2nd}} = C_{ij}^{2nd} f_j^{MC^{2nd}} \\
\vdots \\
f_i^{MC^{final}} = C_{ij}^{final} f_j^{MC^{final}} \]

We can reconstruct the \( x \) distribution using \( C_{ij}^{final} \) as follows.

\[ f_i^{Data} = C_{ij}^{final} f_j^{Data}. \quad (A.6) \]

In this method a smoothing procedure has some arbitrariness and problems in estimating the correct errors in each bin and in the bin-to-bin correlations exist.
Bibliography


[4] ...

[5] ...


TASSO Collab., M. Althoff et al., Z. Phys. C31, 527(1986) ;

[9] V. M. Budnev, I. F. Ginzburg, G. V. Meledin and V. G. Serbo,


[34] “TOPAZ Vertex Chamber and New Inner Drift Chamber”, Proposal to
    KEK, the National Laboratory for high Energy Physics, Japan, KEK-

[35] M. Kobayashi, KEK Preprint 90-22(1990);M. Kobayashi et al., KEK


[46] S. J. Brodsky, T. DeGrand, J. Gunion and J. Weis,
    Phys. Rev. D19, 1418(1979)


List of Figures

1.1 The schematic view of the electron-proton scattering. The Fig. (a) shows elastic electron-proton scattering, (b) shows deep inelastic electron-proton scattering, where the substructure quarks can be seen in this process. (c) If the $Q^2$ is increased, the gluons in the proton appear. ........................................... 8

1.2 The deep inelastic electron photon scattering in $e^+e^-$ reactions. 9

2.1 Deep inelastic electron-photon collisions in two photon process. 12

2.2 Deep inelastic electron-photon scattering. ......................... 13

2.3 $\gamma^*\gamma$ collisions. ........................................ 16

2.4 $\gamma^*\gamma$ collisions in QPM. ...................................... 17

2.5 Feynman diagrams for the photon structure function in QPM. 18

2.6 The corrections of gluon effects for $\gamma^*\gamma$ collisions. (a) shows the correction for the point-like component of the target photon, and (b) shows the correction for the hadronic component. .... 19

2.7 The leading order Altarelli-Parisi splitting functions. ........... 21

2.8 The photon structure function in the DG parametrization (solid line) with PLUTO data. The short dashed curve is the QPM prediction for charm production using $m_c = 1.5$ GeV . ......... 23

2.9 A comparison of the prediction of the LAC parametrization with $Q_0^2 = 1$GeV$^2$(dashed line), with $Q_0^2 = 4$GeV$^2$ (full line) and the DG parametrization (dotted line) with the measurement of $F_2^p$ as a function of $x$ for different values of $Q^2$ ............... 24

2.10 Schematic diagram of the photon structure function in QCD. .. 25
2.11 The photon structure function in FKP formula. (a) show the 
dependence of $p_t$ for $F_{4}^{FKP}$ at the QCD scale parameter $\Lambda = 
0.2$ GeV and (b) shows that of the $\Lambda$ at $p_t = 0.5$ GeV.  

3.1 A plain view of TRISTAN.  
3.2 Integrated luminosity per day.  
3.3 Specific luminosity. (a) shows at higher energy runs and (b) at 
higher luminosity runs.  
3.4 A bird’s eye view of the TOPAZ detector.  
3.5 A cross-sectional view of the TOPAZ detector.  
3.6 A coordinate system of the TOPAZ detector.  
3.7 The structure of the Time Projection Chamber.  
3.8 A sector of the Time Projection chamber.  
3.9 The Gating grid of the Time Projection Chamber. (a) shows 
the open mode and (b) shows the shut mode.  
3.10 Momentum resolution of the Time Projection Chamber as a 
function of $P_t$.  
3.11 The schematic view of the Barrel Calorimeter.  
3.12 The energy distribution for Bhabha events with the Barrel Calorim-
eter.  
3.13 The schematic view of the Luminosity Monitor.  
3.14 One module of the Luminosity Monitor.  
3.15 The energy distribution for Bhabha events with the Luminosity 
Monitor.  
3.16 The energy distribution for Bhabha events with the Endcap 
Calorimeter.  
3.17 A cross-sectional view of the upgraded TOPAZ detector.  
3.18 A sketch of mechanical substructure of the Vertex Chamber.  
3.19 The wire configuration of the Vertex Chamber.  
3.20 The structure of the Forward Calorimeter.  
3.21 The cross-sectional view of Forward Calorimeter.
3.22 The structure of BGO Calorimeter. .......................... 49
3.23 Energy resolution of BGO calorimeter. ....................... 50
3.24 Energy distribution for Bhabha events with the Forward Calorimeter. ........................................ 51
3.25 Structure of the Si-strip detector. ............................... 52
3.26 Pulse height distribution of the Si-strip detector. ............ 53
3.27 Angular distribution for Bhabha events with the Forward Calorimeter. ........................................ 54
3.28 A block diagram of the TOPAZ trigger system. ............... 56
3.29 A block diagram of the Energy Trigger. ....................... 57
3.30 TPC wire configuration and the logic to remove noise. ........ 60
3.31 TPC wire signal generated by a track from the event vertex. 60
3.32 The vertex decision in TPC Trigger. ............................ 61
3.33 The $z$ distribution of the tracks. The solid histogram is the $z$
  distribution of software triggered events and the dashed one is
  that of TPC triggered events. .................................... 62
3.34 The TOPAZ data acquisition system. ........................... 63

4.1 The trigger logic and the timing chart. (a) shows the trigger
  logic for LUM trigger and (b) shows the timing chart for Bhabha
  trigger and (c) for Background trigger. .......................... 67
4.2 Pulse height distribution for Defining and Complementary counters. (a) shows that for $D_1$ and (b) for $C_4$. ............... 68
4.3 The scatter plot of the deposit energy in the $S_1$ and $S_4$
  counters for Bhabha triggered events (a) and for Background triggered
  events (b). .................................................. 69
4.4 Average pulse height of $D_1$, $C_1$ and $S_1$ counters for Bhabha
  events during long time from Jul. 27 in 1987 to Mar. 14 in 1988. 71
4.5 The simulated $\eta_{single}$. .................................... 73
4.6 The Monte Carlo simulation for Bhabha events. ................. 75
4.7 The energy threshold effects for luminosity measurement. .... 76
4.8 The measured the ratio $R(\equiv (N_1 + N_2)/(N_3 + N_4)$}. .......... 77
4.9 The measured luminosity as a function of beam current.  

5.1 The flow of the TOPAZ data processing.  

5.2 A block diagram of the Time Projection Chamber analysis.  

5.3 The local coordinate system $(\xi, \eta, z)$.  

5.4 The parameterization for a charged track of Time Projection Chamber. 

6.1 The effects of each selection criterion for ECL events. The data in the figures have undergone all cuts except the one in question. The figures show (a): vertex $z$ position, (b): visible mass of the hadron system, (c): the sum of the transverse momentum vectors, (d): the visible energy of the hadron system, (e): the total longitudinal momenta and (f): the sum of the absolute value of the momenta around the tagged electron. The cut position is shown by arrow in each figure. 

6.2 The effect of each selection criterion for FCL events. The data in the figures have undergone all cuts except the one in question. The figures show (a): the vertex $z$ position, (b): visible mass of the hadron system, (c): the sum of transverse momentum vectors, (d): the visible energy of the hadron system and (e): the longitudinal momentum distribution. The arrow shown in each figure shows the cut position to select the single-tag two-photon events. 

6.3 The Feynman diagrams of $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ and $e^+e^- \rightarrow e^+e^- q \bar{q}$ process. 

6.4 Inelastic Compton scattering. 

6.5 Hadron production and tau pair production via one photon process. 

7.1 The flow of Monte Carlo data analysis and the flow of experimental data analysis. 

7.2 Outline of single-tag event generation.
7.3 The principle of the Charged Pre-trigger. (a) shows a trigger condition for two tracks of back-to-back condition, and (b) shows for each tracks of \( p_t \) condition.  

7.4 The trigger efficiency in the cosmic ray test.  

7.5 The principle of the TPC trigger.  

7.6 The scatter plot of the longitudinal momentum imbalance vs. the visible energy of the hadron system. The distributions of simulated hadronic events in the one-photon process and \( e^+e^-q\bar{q} \) events are shown in (a) and (b), respectively. That of experimental data are shown in (c).  

7.7 The opening angle distributions between a tagged \( e^\pm \) and the other particles of hadronic events in the one-photon process, \( e^+e^-q\bar{q} \) events and data are shown in (a), (b) and (c). The momentum summing distribution within 0.4 radian around tagging \( e^\pm \) of hadronic events, \( e^+e^-q\bar{q} \) events and data are shown in (d), (e) and (f).  

7.8 The distribution of the sum of transverse momentum vectors. The distributions of simulated \( \tau^+\tau^- \) hadronic events in the one-photon process and \( e^+e^-q\bar{q} \) events are shown in (a), (b) and (c), respectively. That of ECL events are shown in (d).  

7.9 The ECL event distribution of event vertex \( z \) are shown with that of the \( e^+e^-q\bar{q} \) events (solid line) of Monte Carlo simulation.  

7.10 The FCL event distribution of event vertex \( z \) distribution.  

7.11 (a)~(e) Comparison of the visible experimental data with two kinds of theoretical predictions. One is FKP(\( u, d, s \)) + QPM(\( c, b \)) + VMD with cut-off \( p_t^0 = 0.5 \) GeV/c and QCD scale parameter \( \Lambda = 0.2 \) GeV ( shown with the solid line ), and the other is QPM(\( u, d, s \)) + QPM(\( c, b \)) + VMD ( shown with a dashed line histogram ).  

8.1 Examples of the normalized orthogonal functions at \( m_0 = 12 \). (a) shows the normalized orthogonal functions used to represent the solution, and (b) shows the \( p_{m_0+1} \) contribution. 

8.2 Histogram of the generated data and unfolding results. (a) shows the generated data with original function and (b) shows the simulated data of (a) with original function. (c) shows the unfolding results with original function.

9.1 Unfolded photon structure function $F_2^l$ distribution at $Q^2 = 9.7 (\text{GeV}/c)^2$ (a) and at $Q^2 = 87 (\text{GeV}/c)^2$ (b).

9.2 Comparison of the measured $F_2^l$ with the theoretical prediction of QPM+VMD. The solid line shows the QPM+VMD prediction. The hadronic contribution, given by VMD, and heavy quark contributions are shown by dotted and dashed lines, respectively.

9.3 Comparison of the measured $F_2^l$ with the theoretical predictions of DG and LAC. The solid line shows the parametrization by DG with $\Lambda = 0.4 \text{ GeV}^2$. LAC's parametrizations with $\Lambda = 0.2 \text{ GeV}^2$ of Set 1, 2 and 3 are shown in the dashed, dotted and dash-dotted line, respectively. $Q_0^2$'s of Set 1, 2 and 3 are 4, 4 and 1 GeV$^2$, respectively. The gluon distribution is different between Sets 1 and 2.

9.4 Comparison of the measured value of $F_2^l$ with the theoretical predictions by FKP($u, d, s$)+QPM($c, b$)+VMD with $p_T^0 = 0.1$, 0.5 and 1 GeV/$c$ at QCD scale parameter $\Lambda = 0.2 \text{ GeV}$. The hadronic contribution given by VMD and that of the heavy quark by QPM are shown in the dotted lines.

9.5 $Q^2$-dependence of the photon structure function $F_2^l$ with previously measured data. The dashed, solid and dotted line shows the theoretical expectation of FKP+VMD with $p_T^0 = 0.1, 0.5$ and 1 GeV/$c$, respectively. The dash-dotted line shows that of QPM+VMD. The hadronic contribution given by VMD is shown in long-dashed line.